So you think you can FEC?

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8th GStreamer Conference
21 October 2017
Prague, Czech Republic

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Outline

- Introduction to FEC
- Designing a FEC decoder element
- FEC algorithms
  - XOR-based FEC
  - Reed-Solomon FEC
- DEMO!

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Bird’s eye view on FEC

RTP video or audio packets (VP8, H264, OPUS, etc)

FEC packets with redundant data

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Bird’s eye view on FEC

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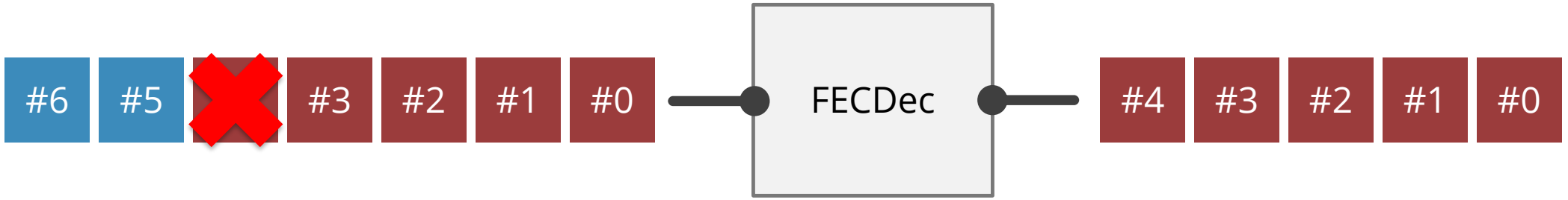
FEC standards zoo

- RED - Redundant Audio Data (RFC 2198)
- ULPFEC based standards:
  - ULPFEC – Generic Forward Error Correction (RFC 5109)
  - ULPFEC (WebRTC)
  - X-ULPFECUC (Skype)
- FLEX FEC - Flexible Forward Error Correction

GStreamer element for packet recovery



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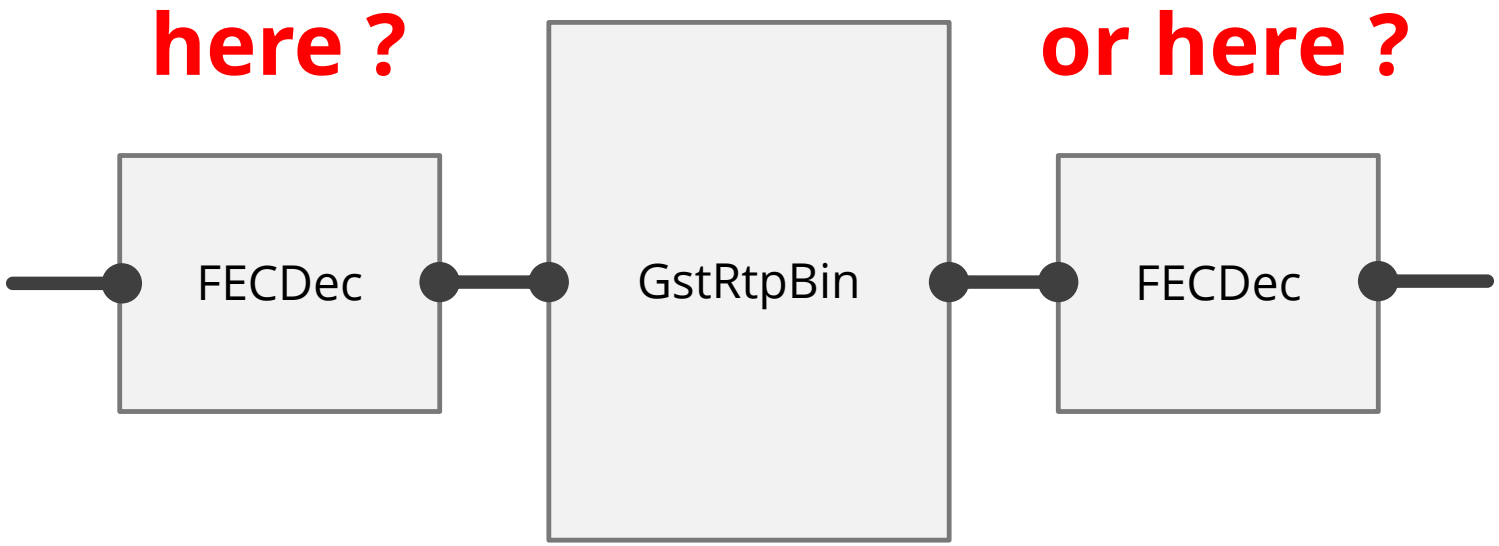
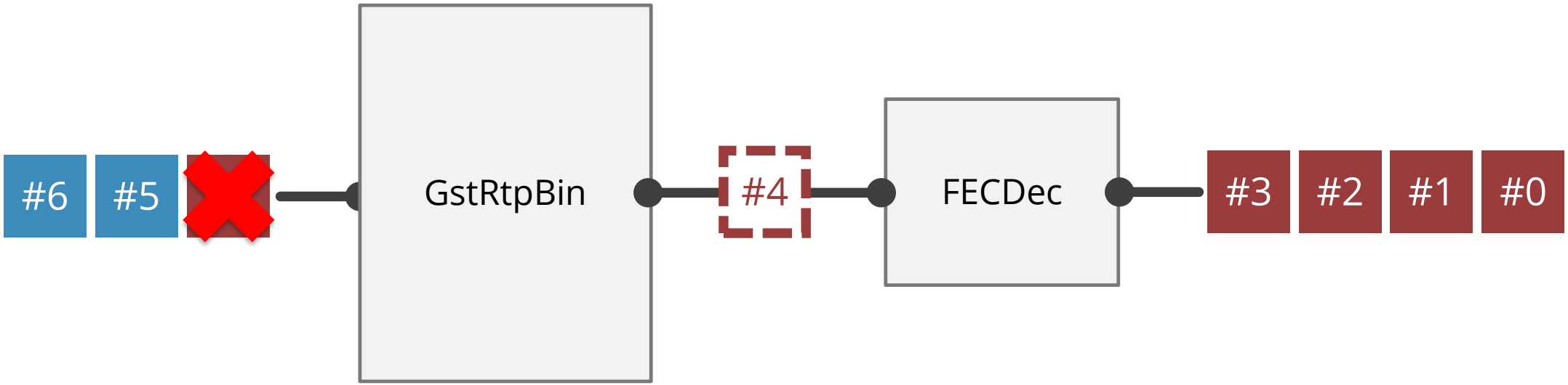
Where to hook it up? Before or after jitterbuffer?

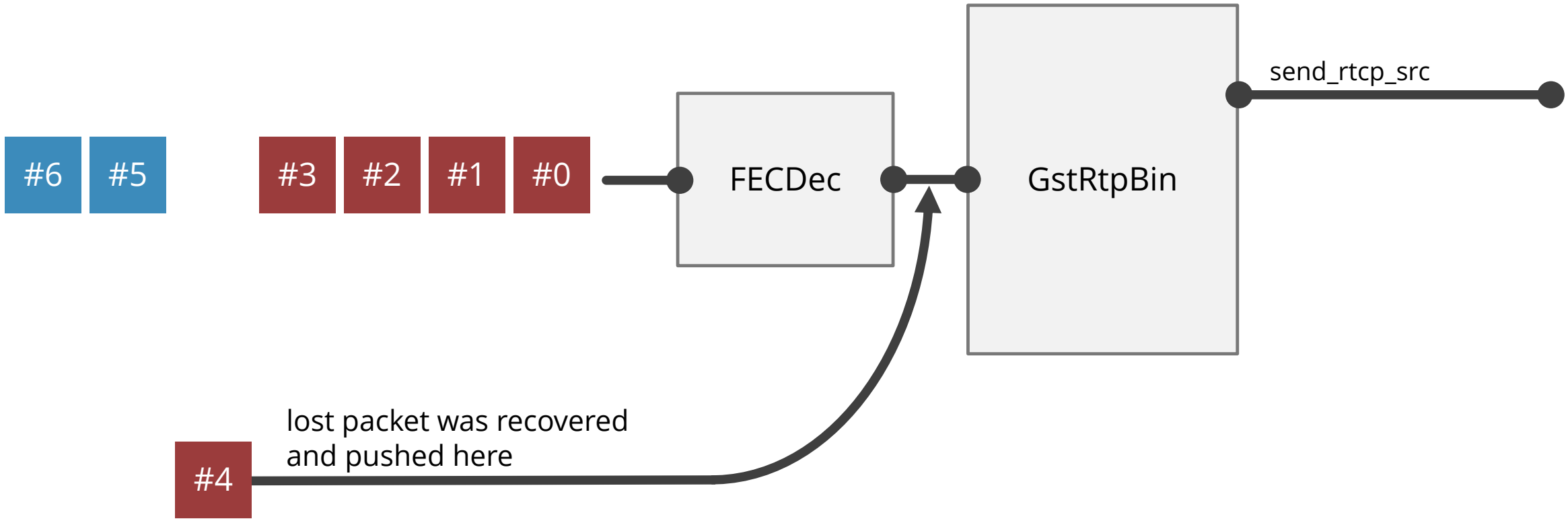


Where to hook it up?

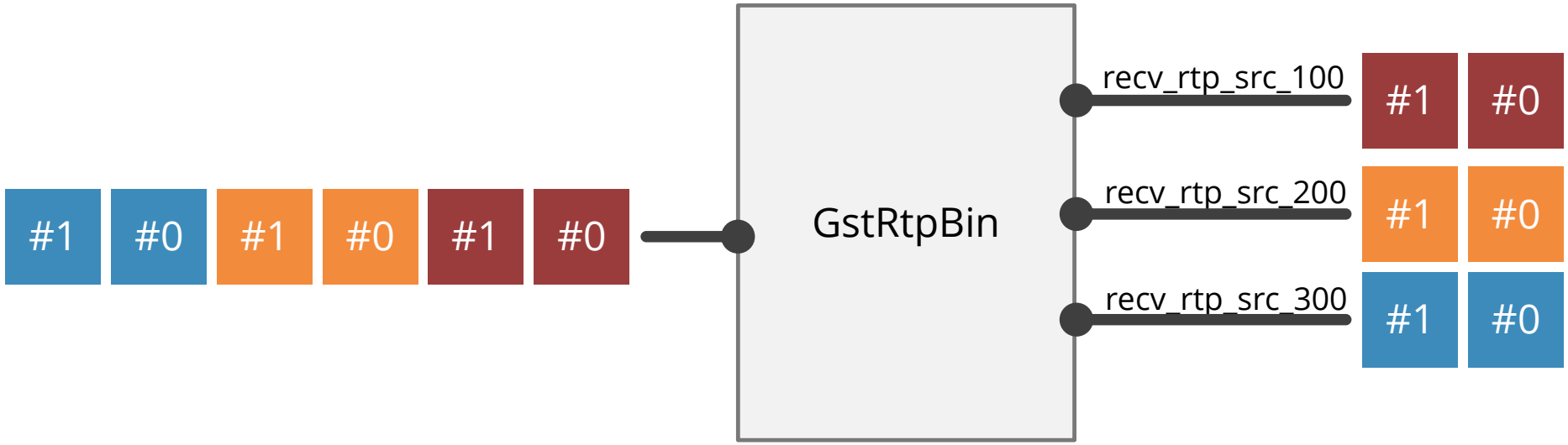


#4 Lost event with seq-num #4

“What if…” it works and recovers packets! Yaaay!



Where to hook it up to? Flex FEC case

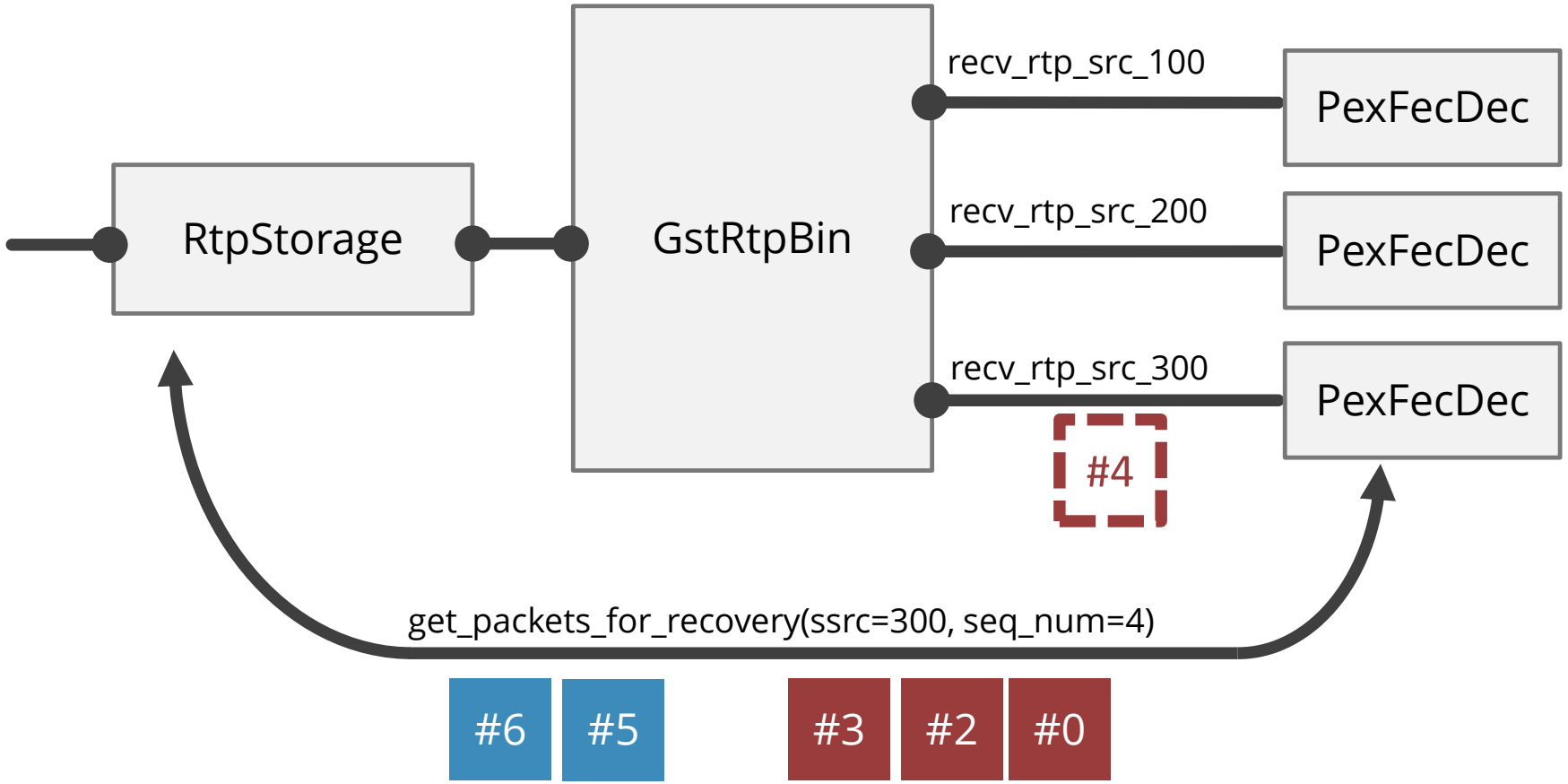


audio packet with ssrc 100

video packet with ssrc 200

Flex FEC packet with ssrc 300

Doing it the right way



Yet another “what if…”

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FEC Algorithms

XOR-based FEC

Reed-Solomon FEC

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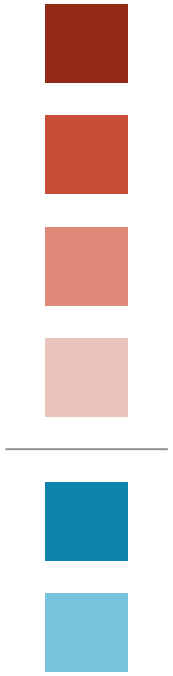
XOR-based FEC



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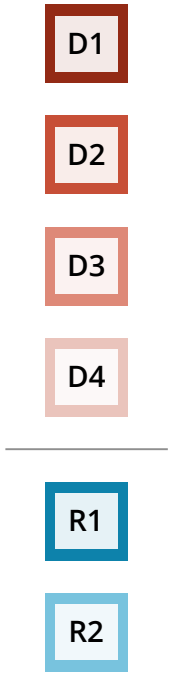
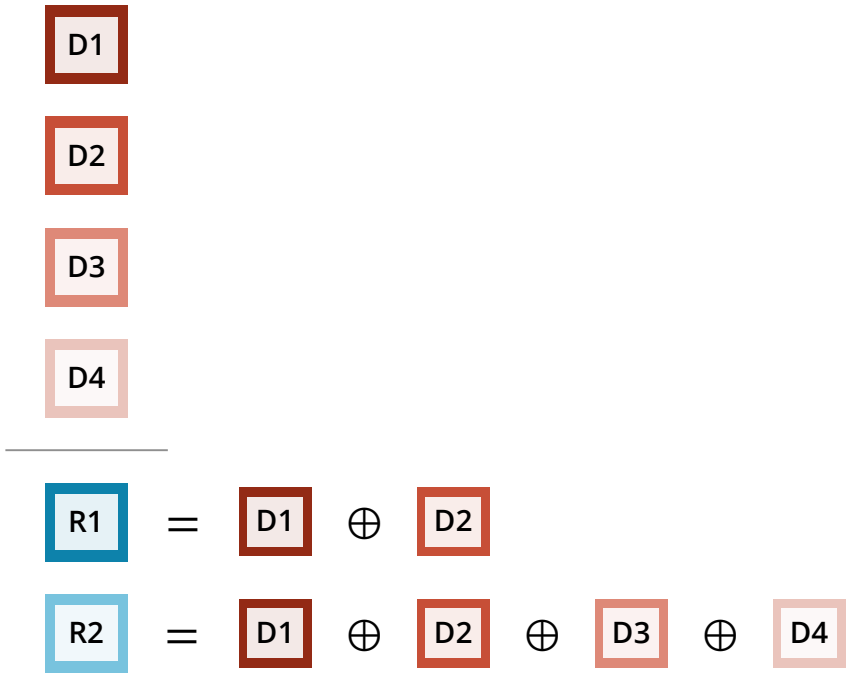
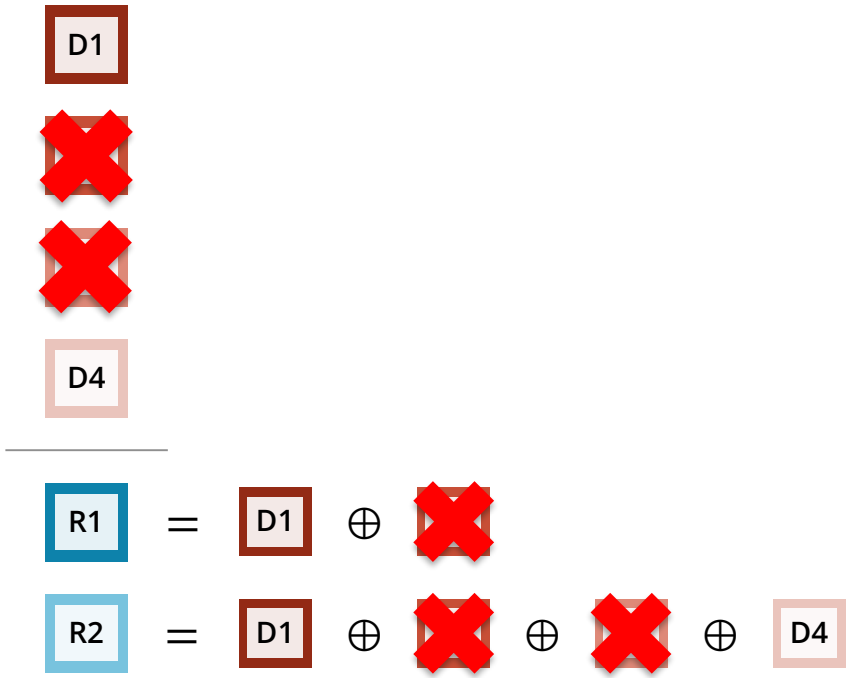
XOR-based FEC



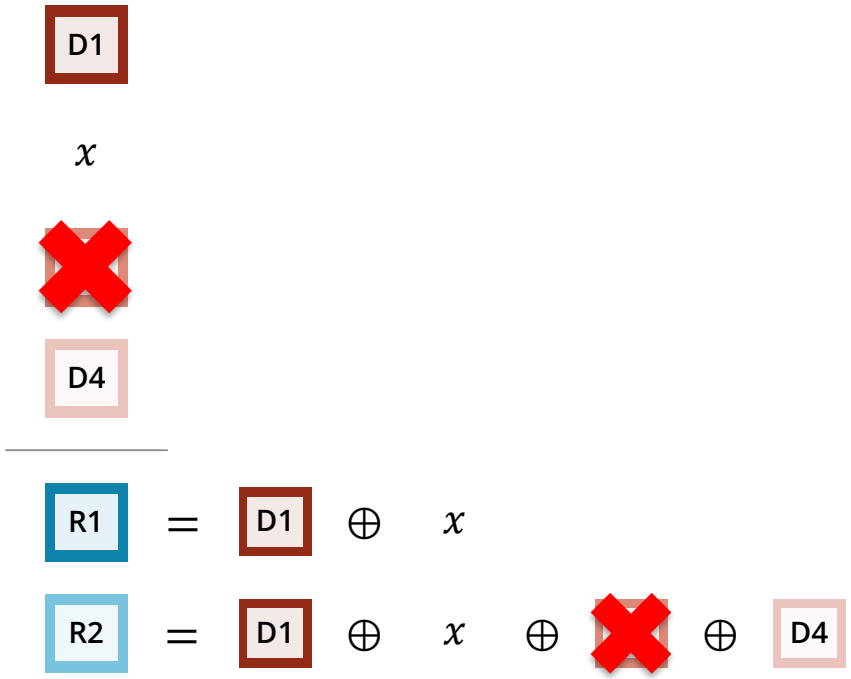
XOR-based FEC



XOR-based FEC



XOR-based FEC



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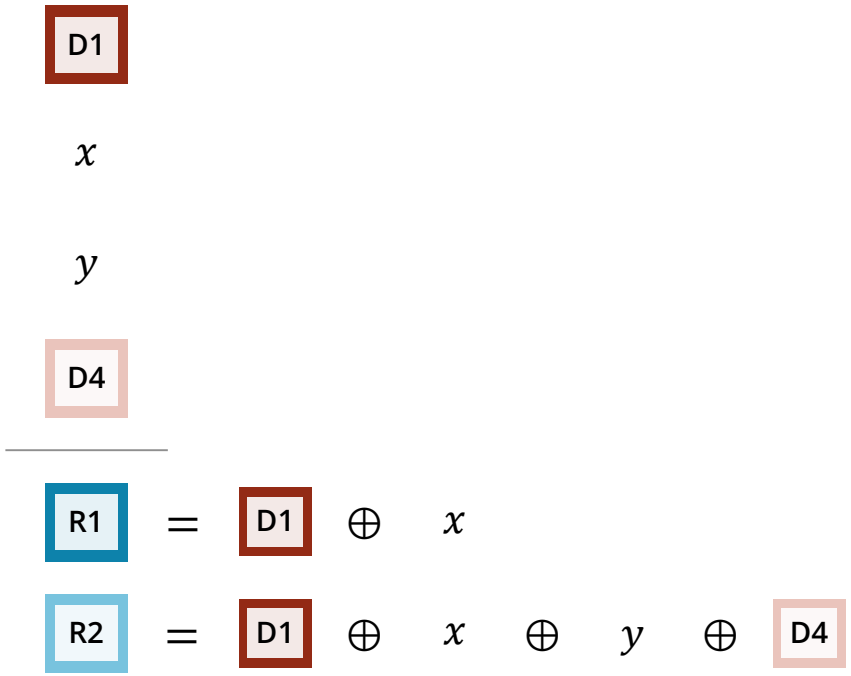

XOR-based FEC

D1

x

y

D4

R1 $=$ D1 $\oplus$ x

R2 $=$ D1 $\oplus$ x $\oplus$ y $\oplus$ D4

$\Leftrightarrow$

x $=$ D1 $\oplus$ R1

y $=$ D1 $\oplus$ x $\oplus$ D4 $\oplus$ R2

R1

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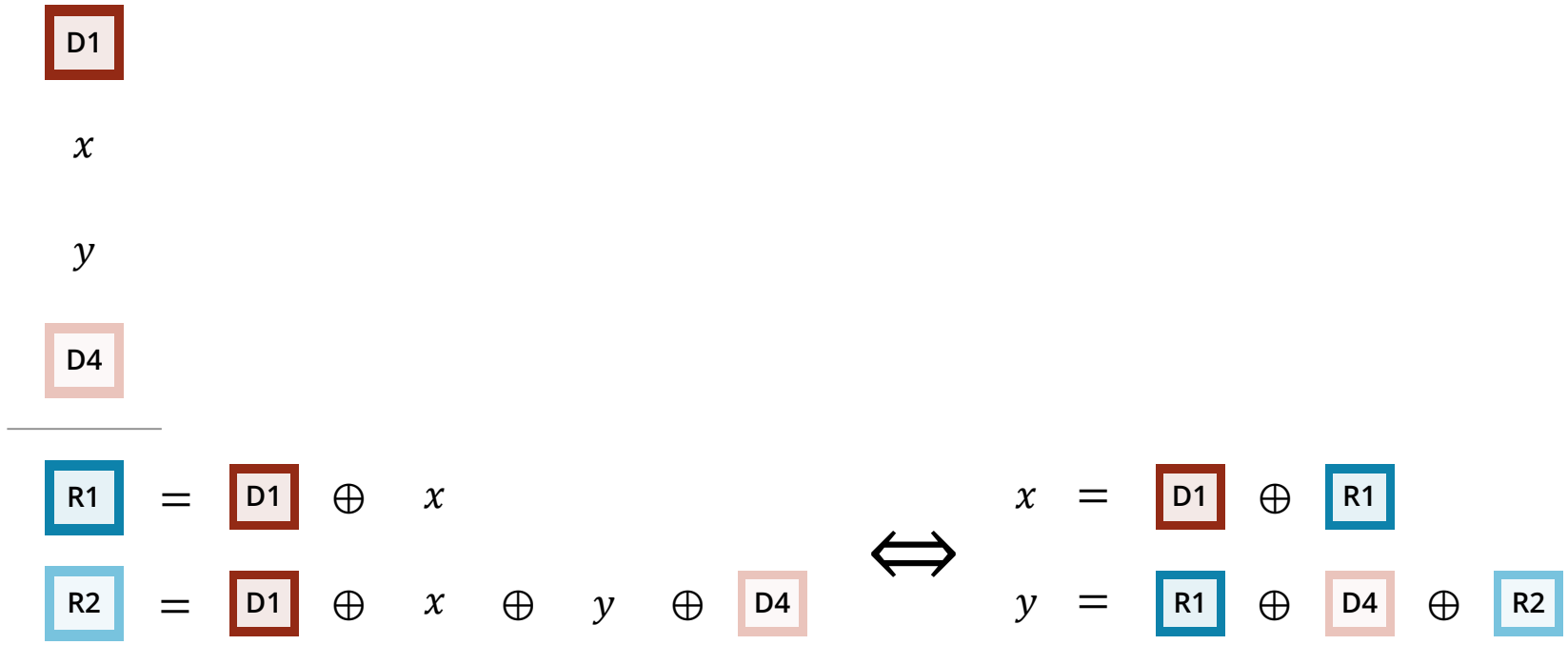
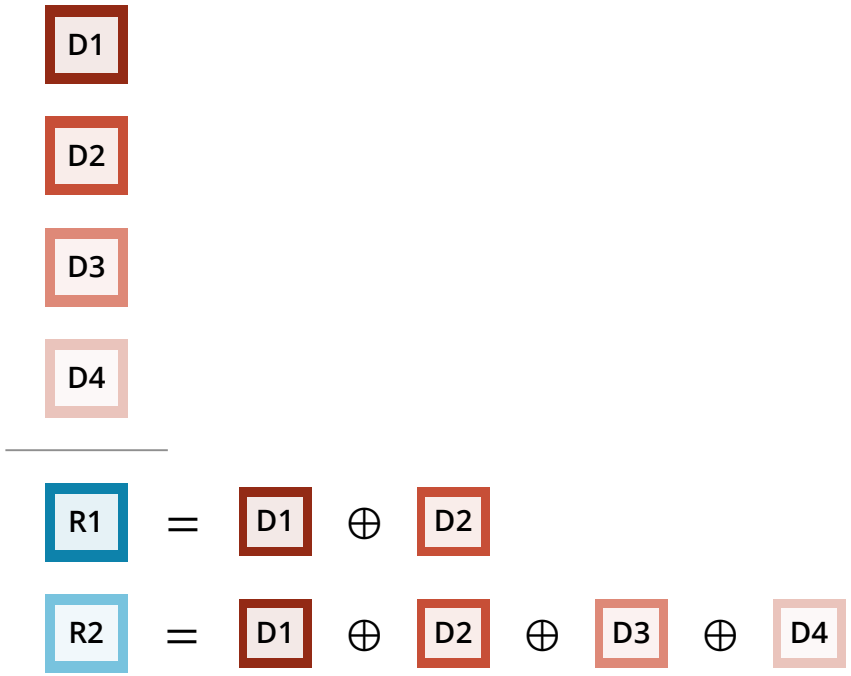
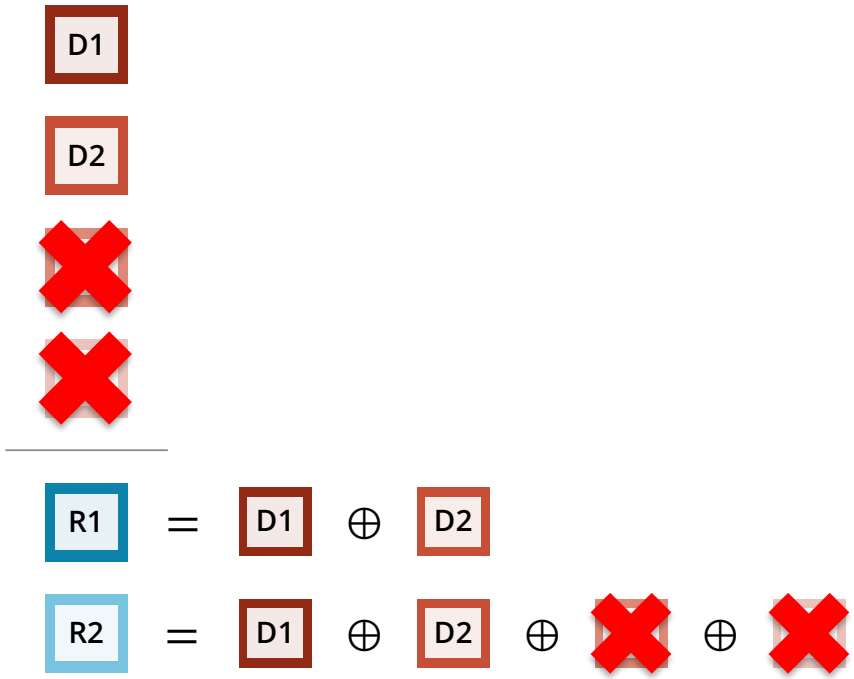
XOR-based FEC



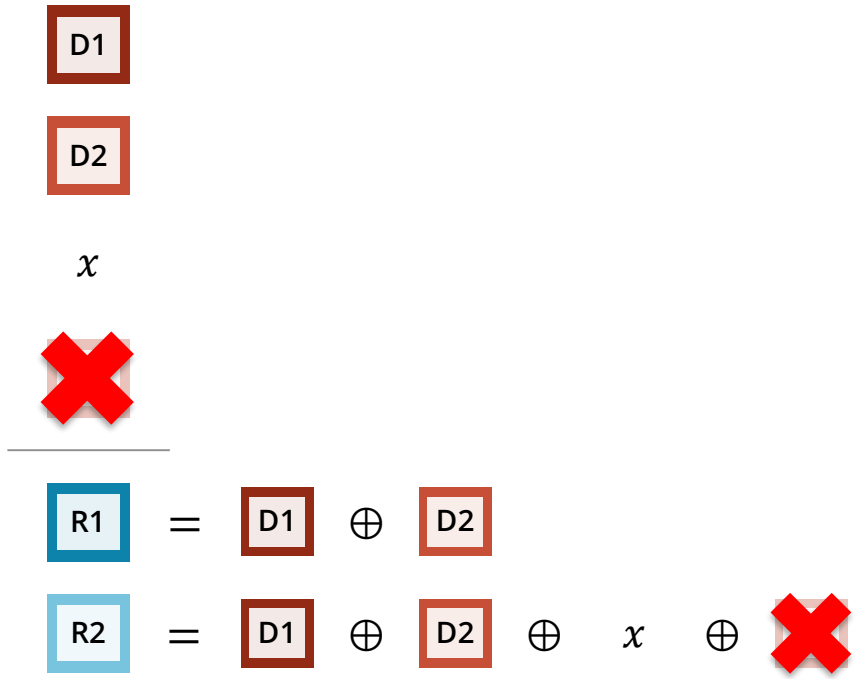
XOR-based FEC



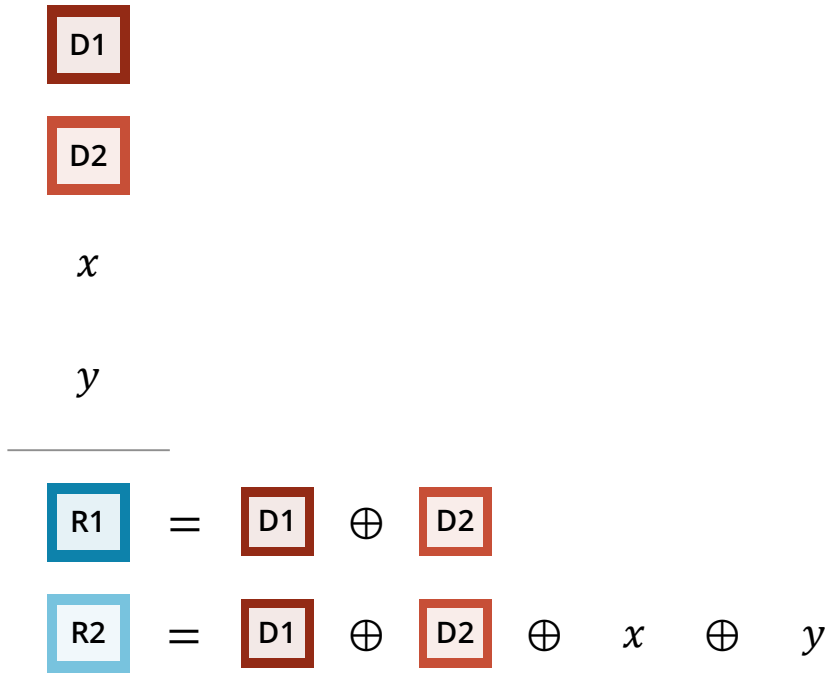
XOR-based FEC



XOR-based FEC

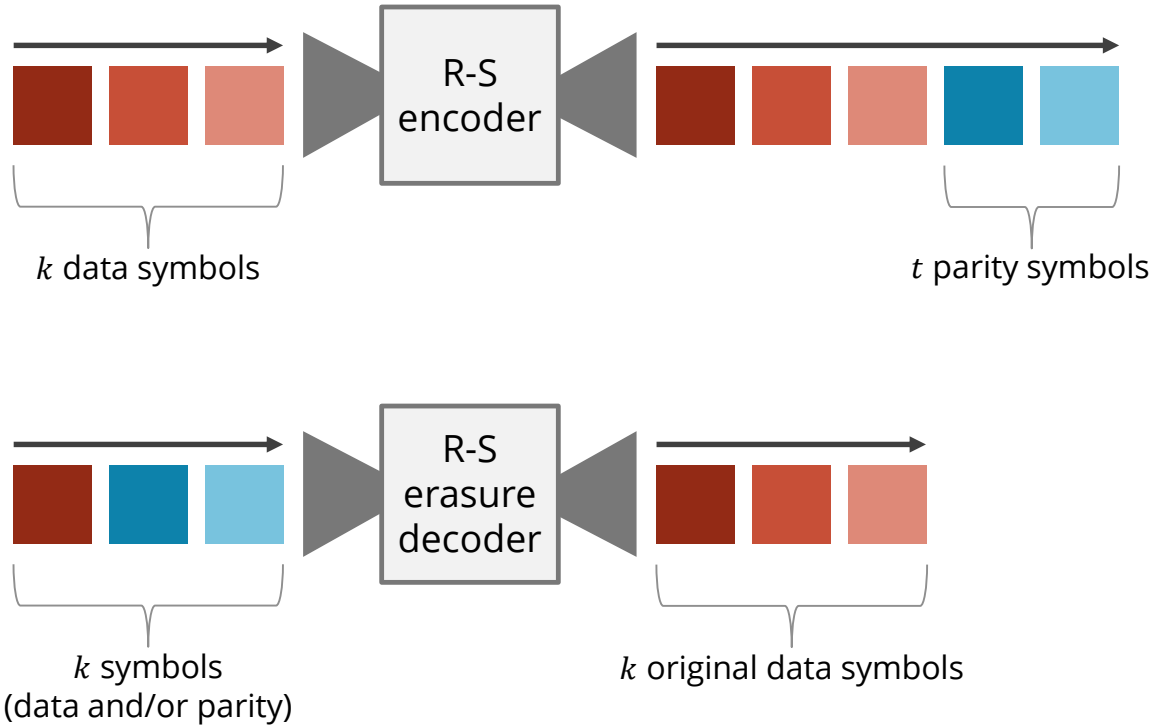


XOR-based FEC



Reed-Solomon codes

Irving S. Reed and Gustave Solomon



- Either
  - Detect up to t errors
  - Detect and correct up to $\lfloor t/2 \rfloor$ errors
  - Correct up to t erasures

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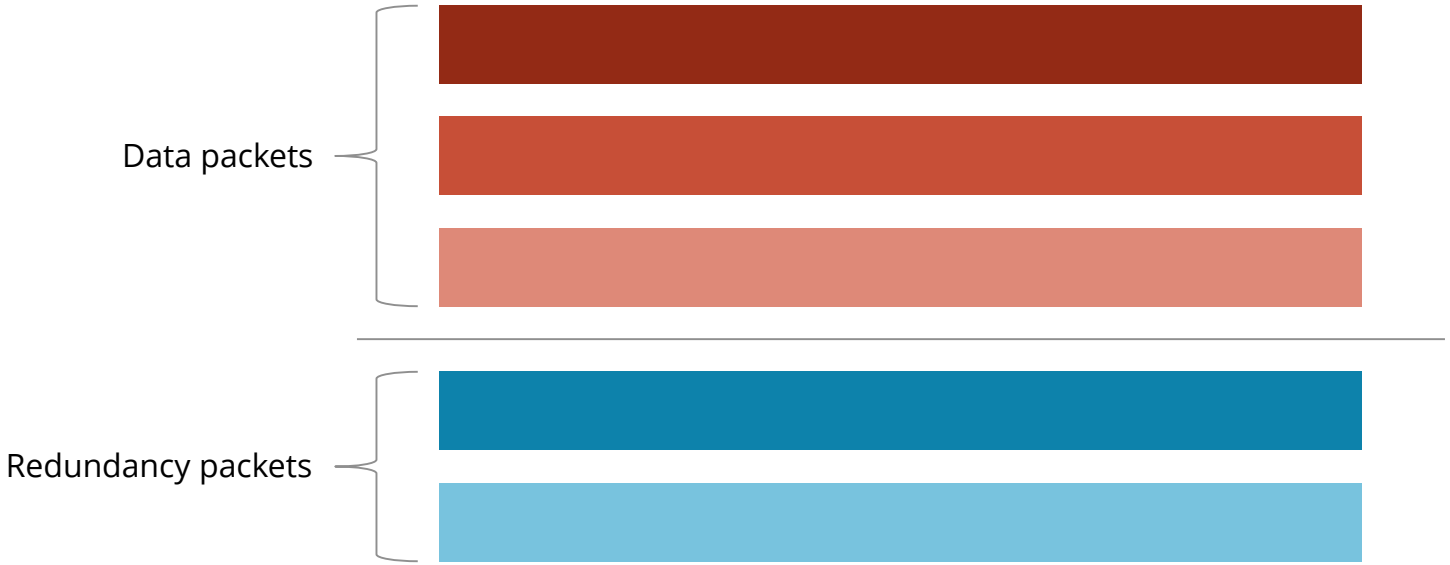
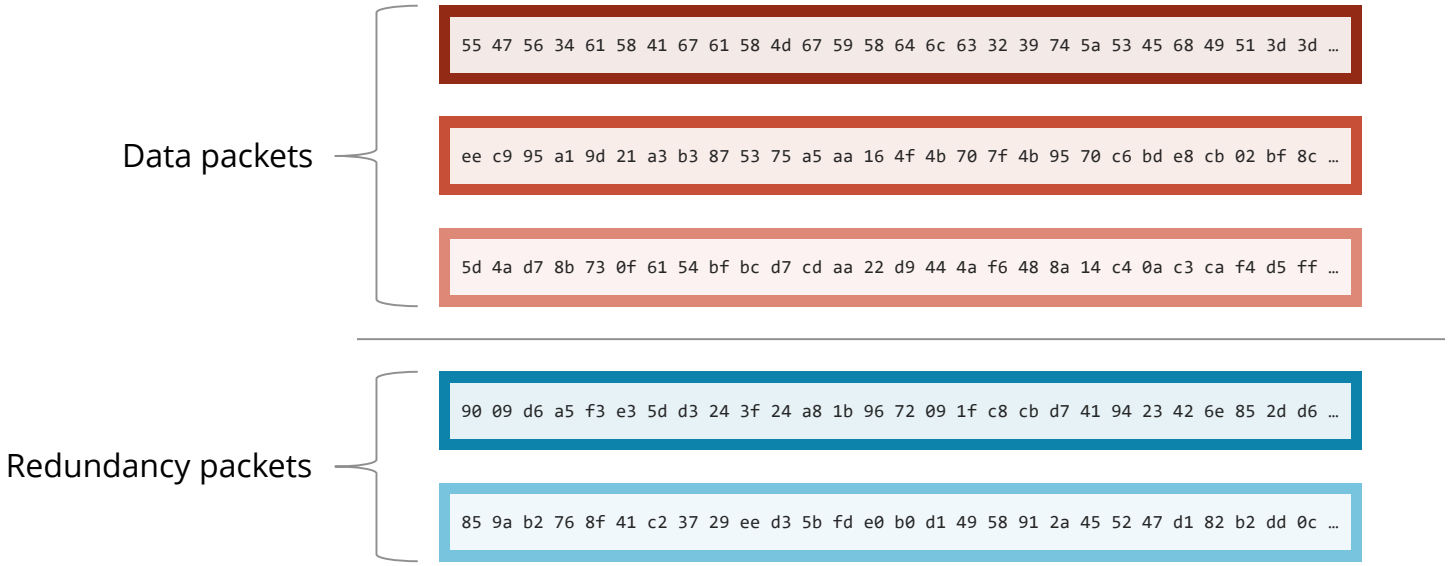
Reed-Solomon codes



Reed-Solomon codes



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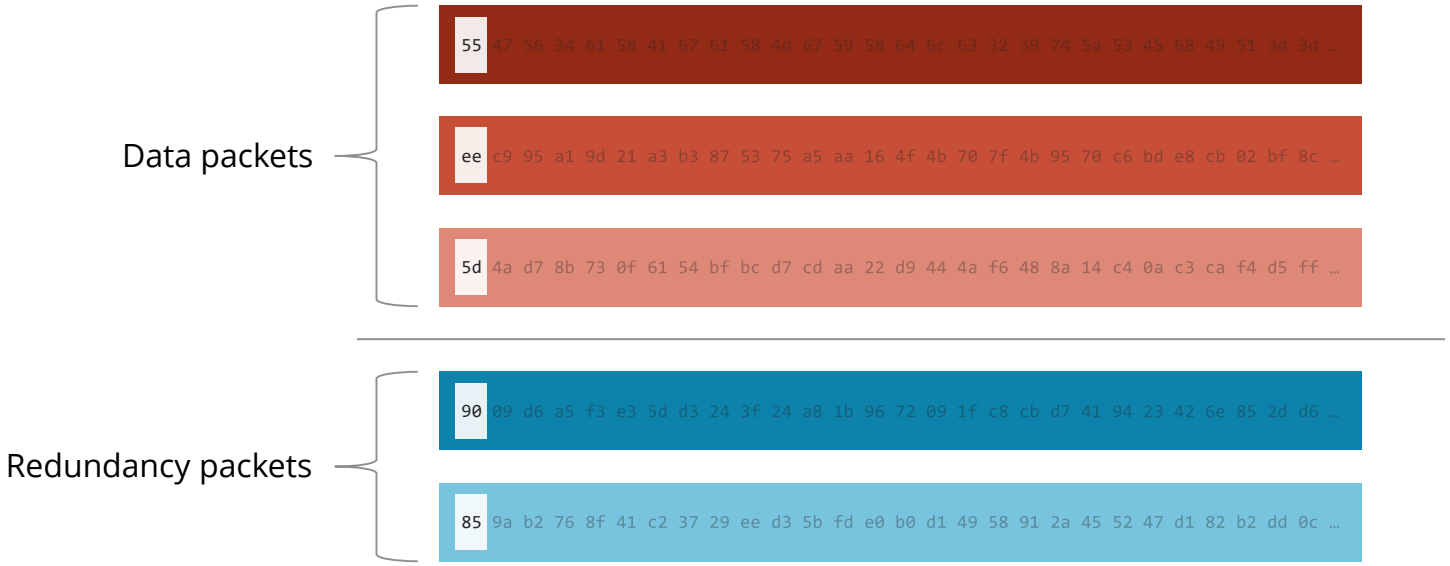
Reed-Solomon codes



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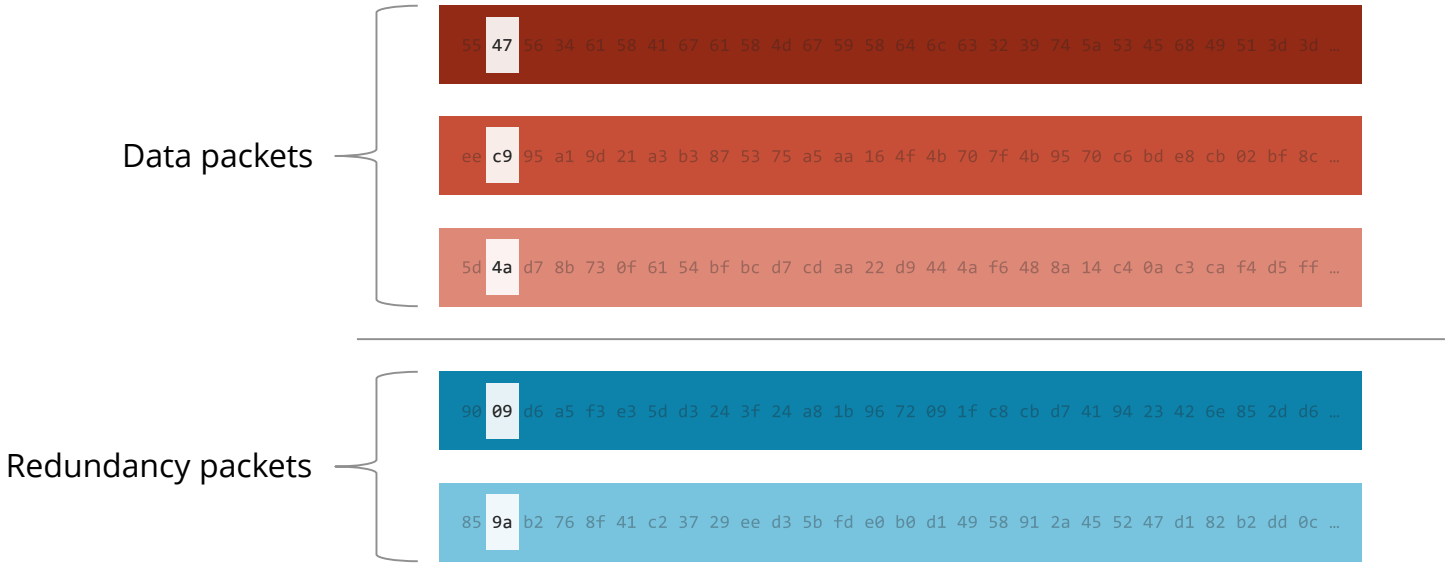
Reed-Solomon codes



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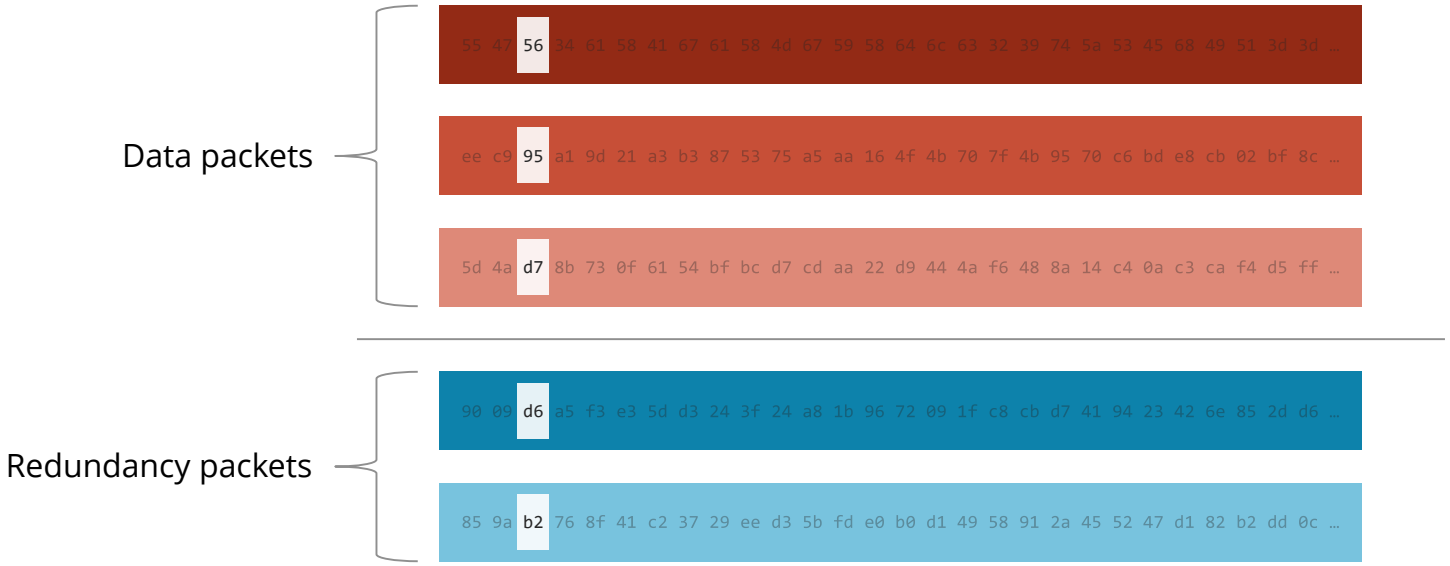
Reed-Solomon codes



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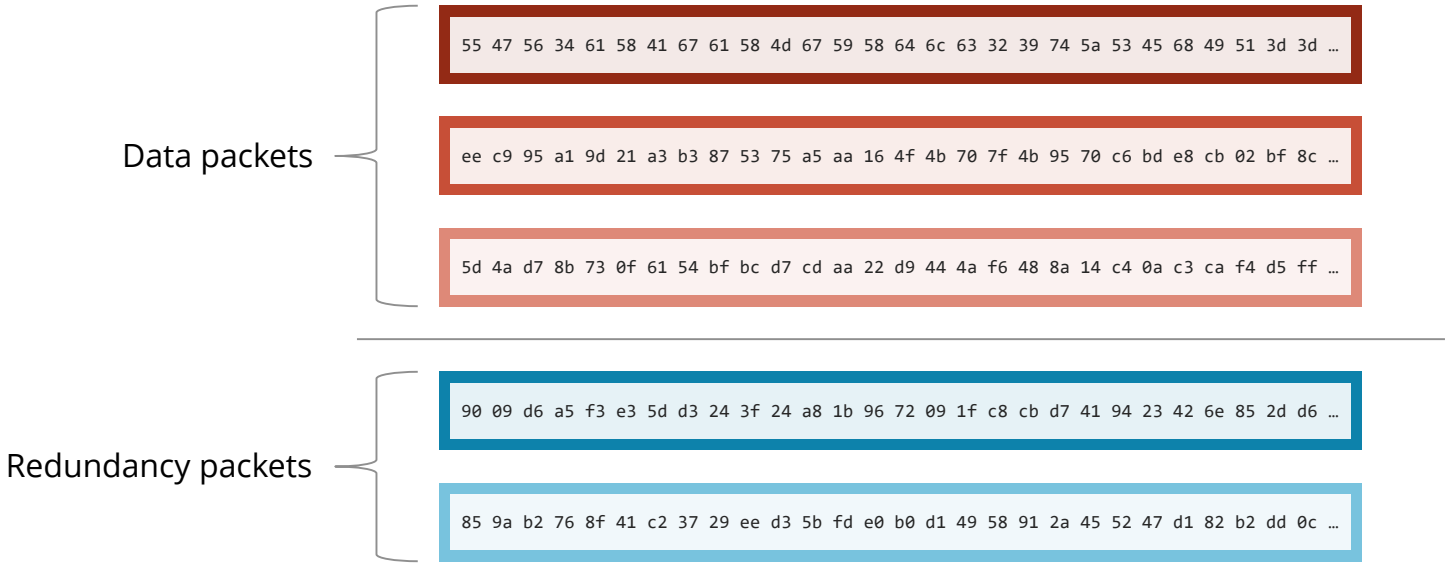
Reed-Solomon codes



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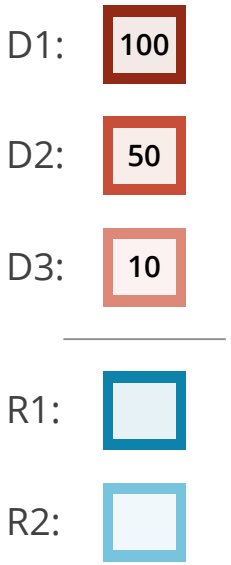
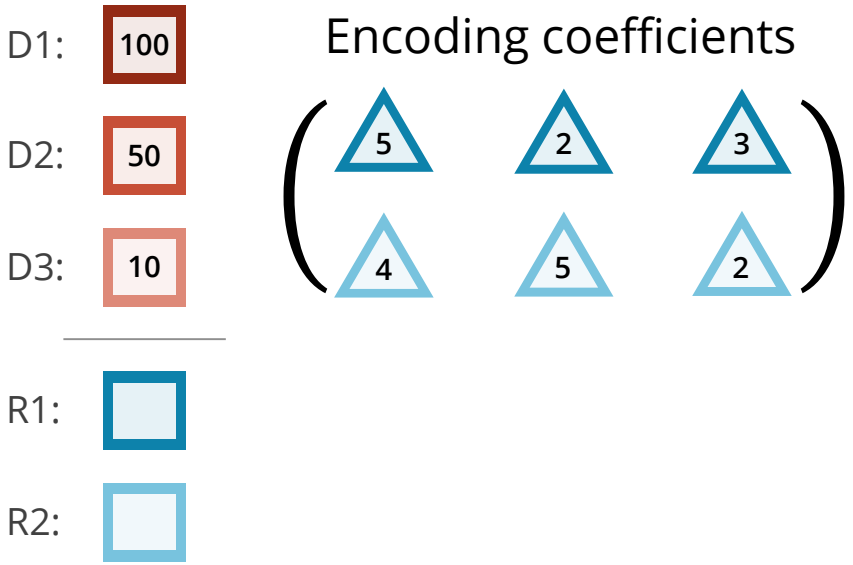
Reed-Solomon codes

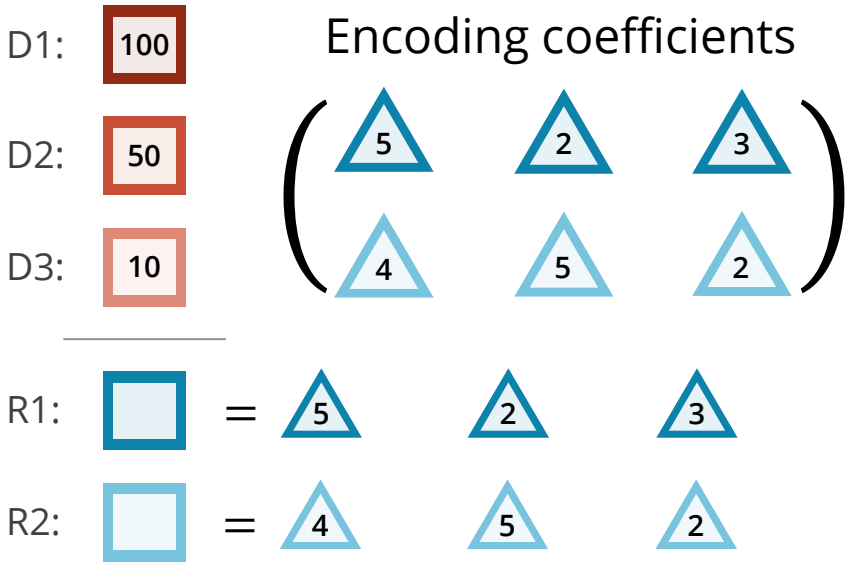
Reed-Solomon codes – encoding



Reed-Solomon codes – encoding



Reed-Solomon codes – encoding



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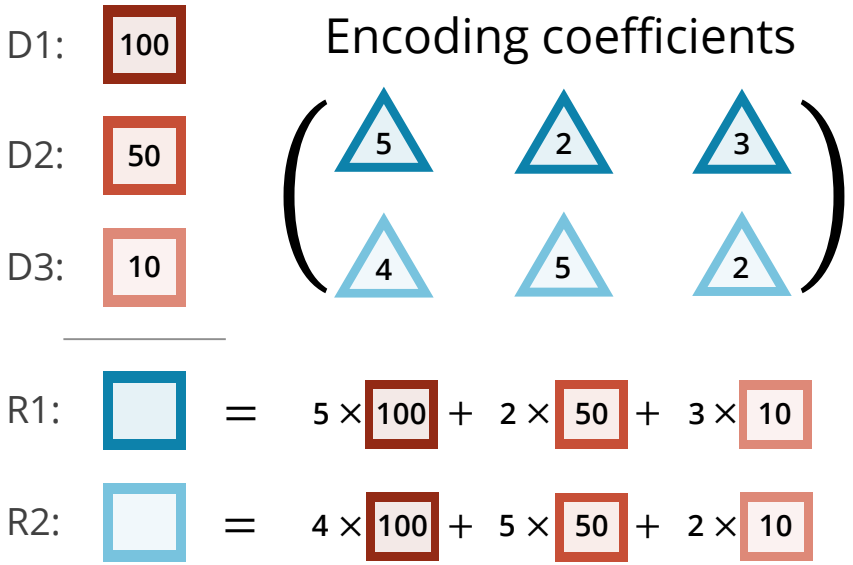

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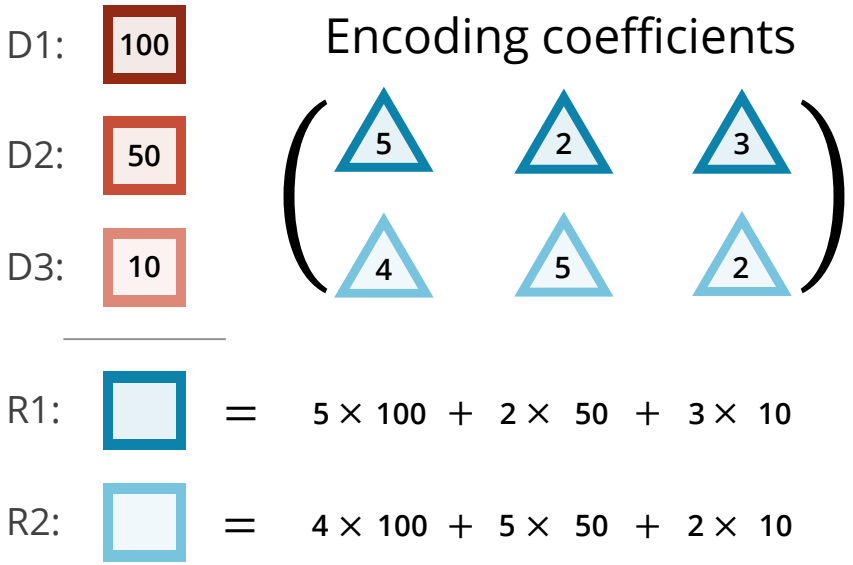
Reed-Solomon codes – encoding



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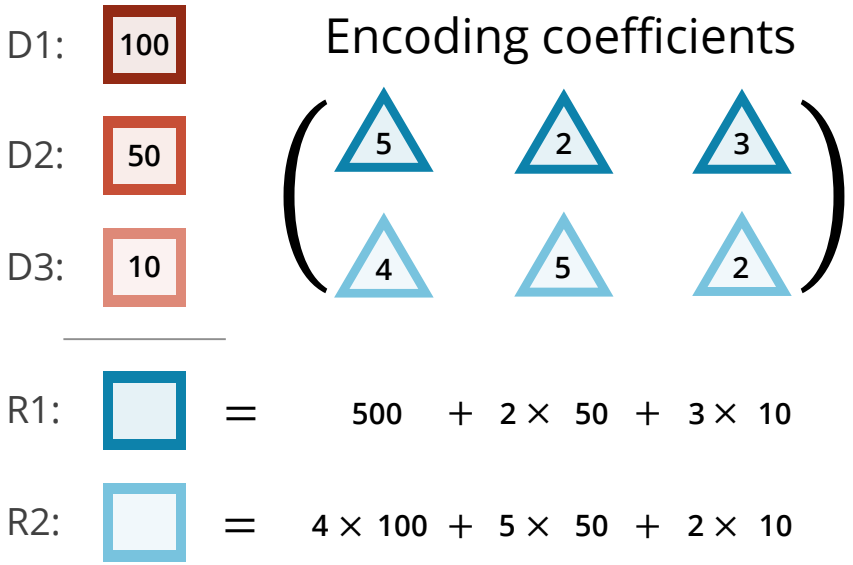

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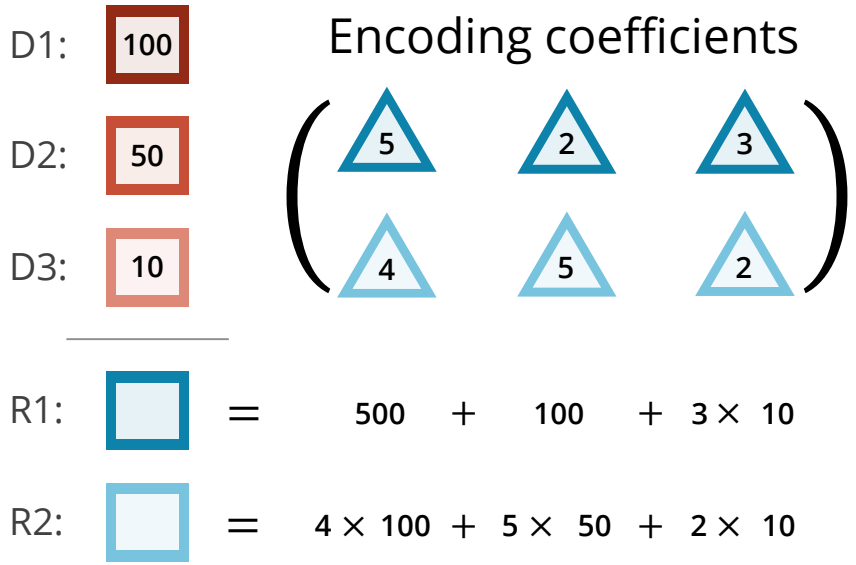
Reed-Solomon codes – encoding



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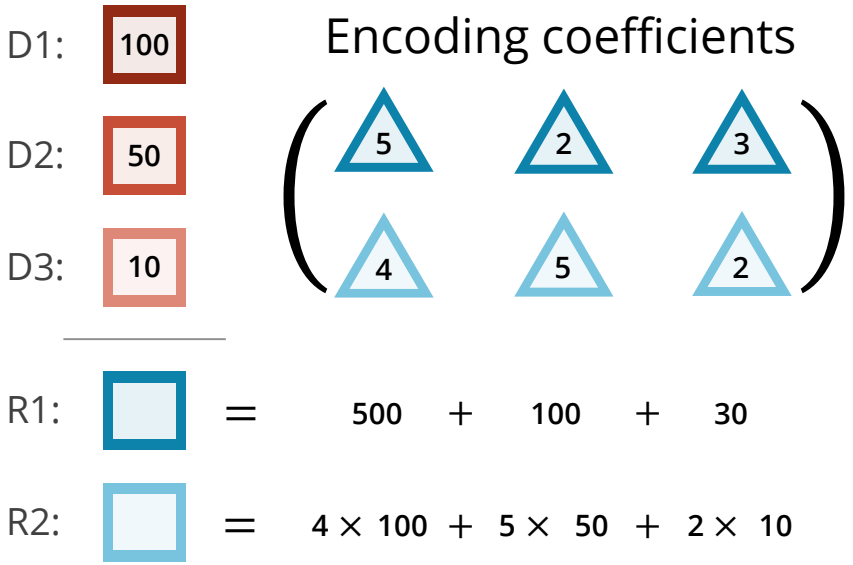
Reed-Solomon codes – encoding



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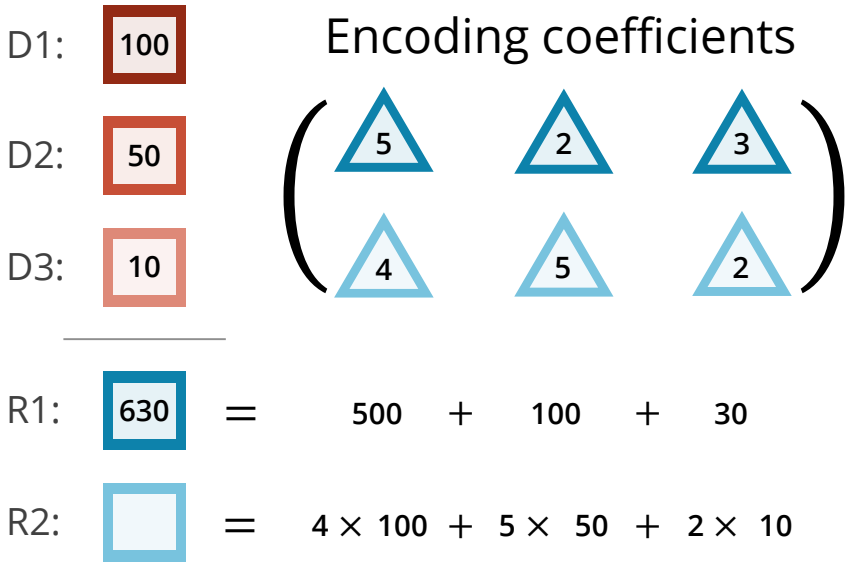
Reed-Solomon codes – encoding



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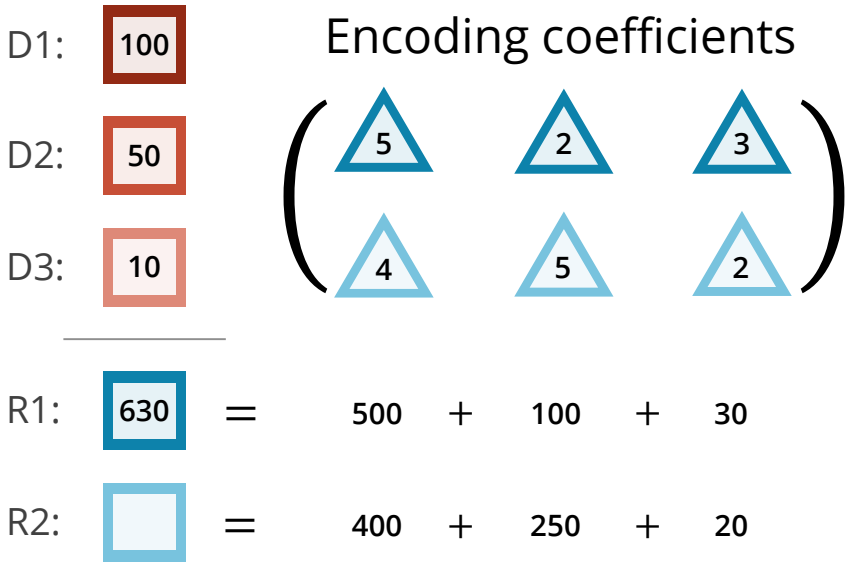
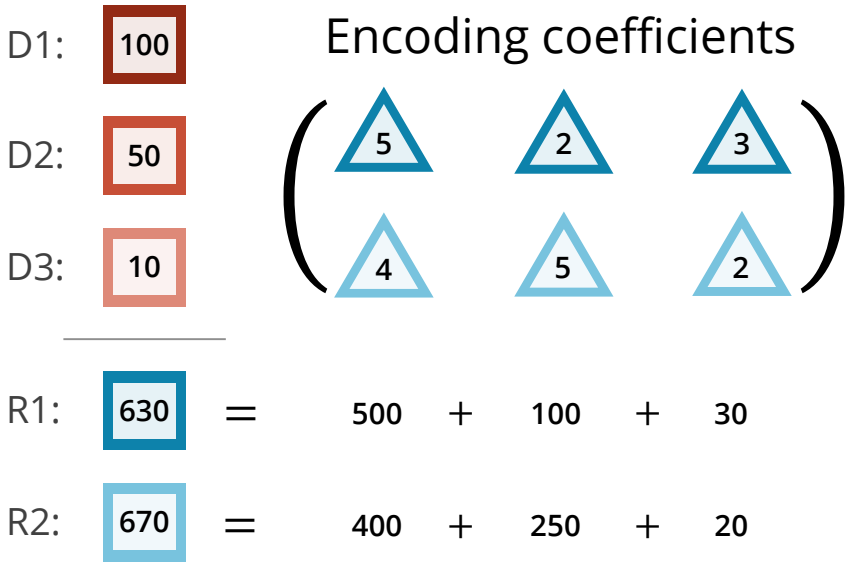
Reed-Solomon codes – encoding



Reed-Solomon codes – encoding



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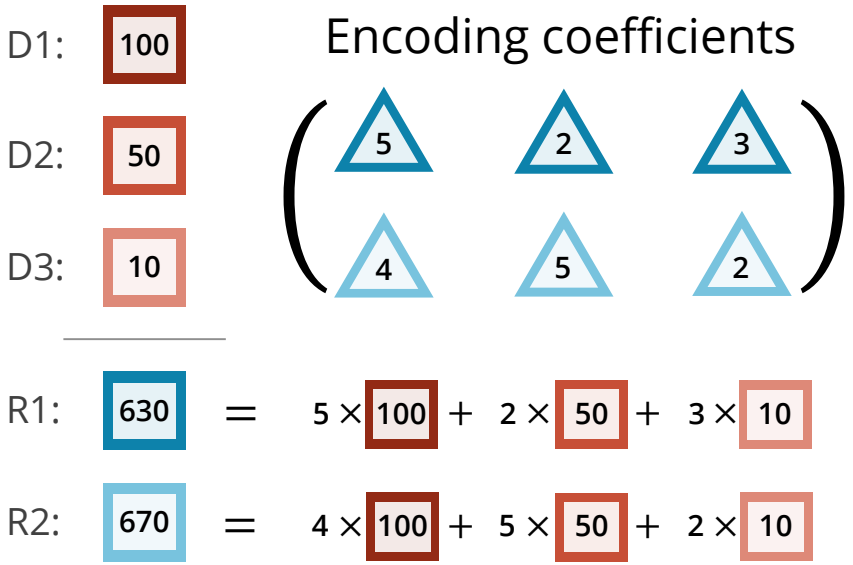
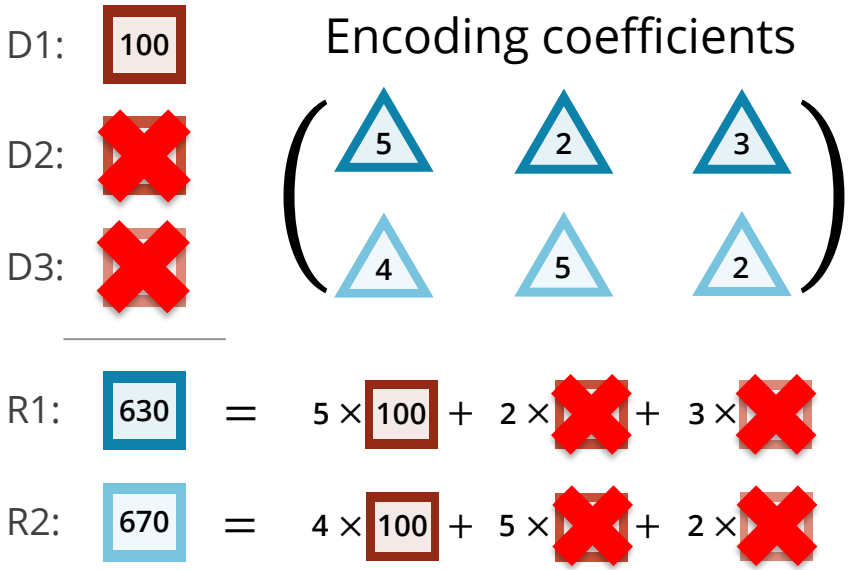
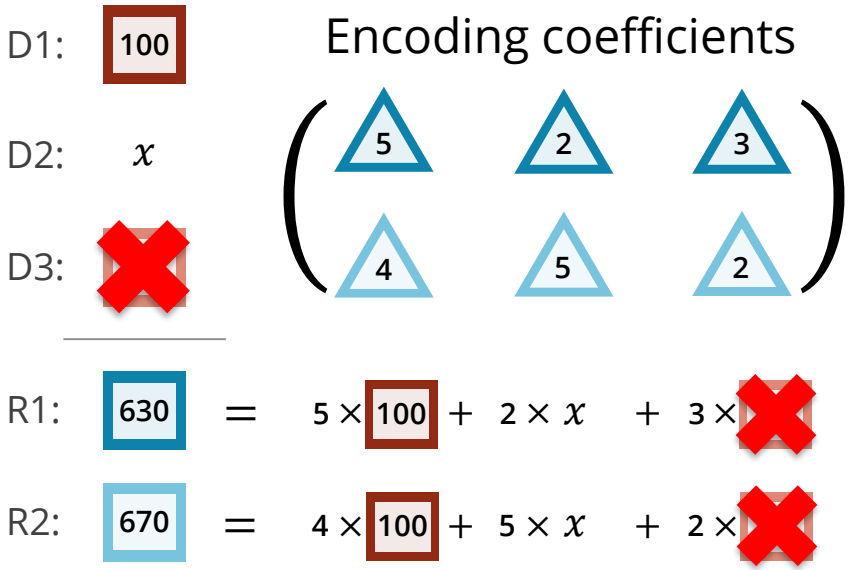
Reed-Solomon codes – decoding



Reed-Solomon codes – decoding



Reed-Solomon codes – decoding



Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: $630 = 5 \times 100 + 2 \times x + 3 \times y$

R2: $670 = 4 \times 100 + 5 \times x + 2 \times y$

Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 630 $= 5 \times 100 + 2 \times x + 3 \times y$

R2: 670 $= 4 \times 100 + 5 \times x + 2 \times y$

$\Leftrightarrow$

$$500 + 2x + 3y = 630$$
$$400 + 5x + 2y = 670$$

Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 630 $= 5 \times 100 + 2 \times x + 3 \times y$

R2: 670 $= 4 \times 100 + 5 \times x + 2 \times y$

$$\Leftrightarrow \begin{cases} \cancel{500} + 2x + 3y = 630 - 500 \\ \cancel{400} + 5x + 2y = 670 - 400 \end{cases}$$

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Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 630 $= 5 \times 100 + 2 \times x + 3 \times y$

R2: 670 $= 4 \times 100 + 5 \times x + 2 \times y$

$\Leftrightarrow$

$$2x + 3y = 630 - 500$$

$$5x + 2y = 670 - 400$$

Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 630 $= 5 \times 100 + 2 \times x + 3 \times y$

R2: 670 $= 4 \times 100 + 5 \times x + 2 \times y$

$\Leftrightarrow$

$$2x + 3y = 130$$

$$5x + 2y = 270$$

Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 630 $= 5 \times 100 + 2 \times x + 3 \times y$

R2: 670 $= 4 \times 100 + 5 \times x + 2 \times y$

$$\Leftrightarrow \quad \begin{aligned} 2x + 3y &= 130 \\ 5x + 2y &= 270 \\ \hline -3x + y &= -140 \end{aligned}$$

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Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 630 $= 5 \times 100 + 2 \times x + 3 \times y$

R2: 670 $= 4 \times 100 + 5 \times x + 2 \times y$

$$\Leftrightarrow \quad \begin{aligned} 2x + 3y &= 130 \\ 5x + 2y &= 270 \end{aligned}$$

$$y = 3x - 140$$

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Reed-Solomon codes – decoding

D1: 100

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 630 $= 5 \times 100 + 2 \times x + 3 \times y$

R2: 670 $= 4 \times 100 + 5 \times x + 2 \times y$

$$\Leftrightarrow \quad \begin{aligned} 2x + 3y &= 130 \\ 5x + 2y &= 270 \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} 2x + 3(3x - 140) &= 130 \\ 2x + 9x - 420 &= 130 \\ 11x &= 550 \\ x &= 50 \end{aligned}$$

$$y = 3x - 140$$

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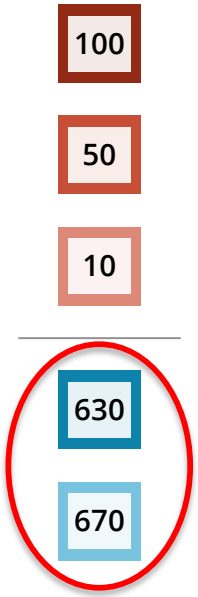
Reed-Solomon codes – summary

- Encoding
  - Linear combination (matrix multiplication)
- Decoding
  - Solving linear equation system (matrix inversion and multiplication)

But...



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Linear combinations in $\mathbb{R}$

$$\underbrace{5 \times 100}_{\in \mathbb{R}} + \underbrace{2 \times 50}_{\in \mathbb{R}} + \underbrace{3 \times 10}_{\in \mathbb{R}}$$

Linear combinations... not in $\mathbb{R}$?

$$\underbrace{5 \times 100}_{\in [0,255]} + \underbrace{2 \times 50}_{\in [0,255]} + \underbrace{3 \times 10}_{\in [0,255]}$$

Linear combinations... not in $\mathbb{R}$?

$$\underbrace{\underbrace{5 \times 100}_{\in [0,255]} + \underbrace{2 \times 50}_{\in [0,255]} + \underbrace{3 \times 10}_{\in [0,255]}}_{\in [0,255]}$$

Fields

- Set of elements
  - $+$ and $\cdot$
  - Given $a \in \mathrm{F}$ and $b \in \mathrm{F}$, then...
    - $(a + b) \in \mathrm{F}$
    - $(a \cdot b) \in \mathrm{F}$
- $\mathbb{R}$ and $\mathbb{Q}$ – infinite fields
- Finite fields

Fields

$$\mathbb{Z}/7\mathbb{Z}$$

$\{0,1,2,3,4,5,6\}$

$+$ is addition modulo 7

$\cdot$ is multiplication modulo 7

Fields

Multiplication table for $\mathbb{Z}/7\mathbb{Z}$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2		4	6	1	3	5
3			2	5	1	4
4				2	6	3
5					4	2
6						1

Fields

Multiplication table for $\mathbb{Z}/_{7\mathbb{Z}}$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2		4	6	1	3	5
3			2	5	1	4
4				2	6	3
5					4	2
6						1

$$2 \cdot 4 \equiv 8 \equiv 1 \pmod 7$$

Fields

Multiplication table for $\mathbb{Z}/_{7\mathbb{Z}}$

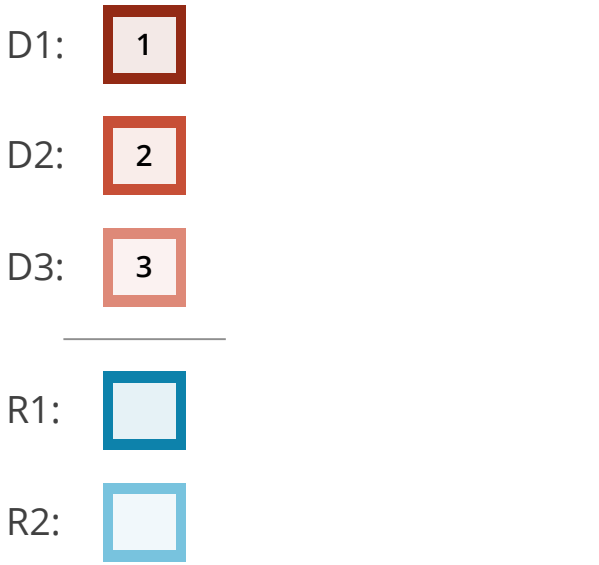
	1	2	3	4	5	6
1	1	2	3	4	5	6
2		4	6	1	3	5
3			2	5	1	4
4				2	6	3
5					4	2
6						1

$$4 \cdot 5 \equiv 20 \equiv 6 \pmod 7$$

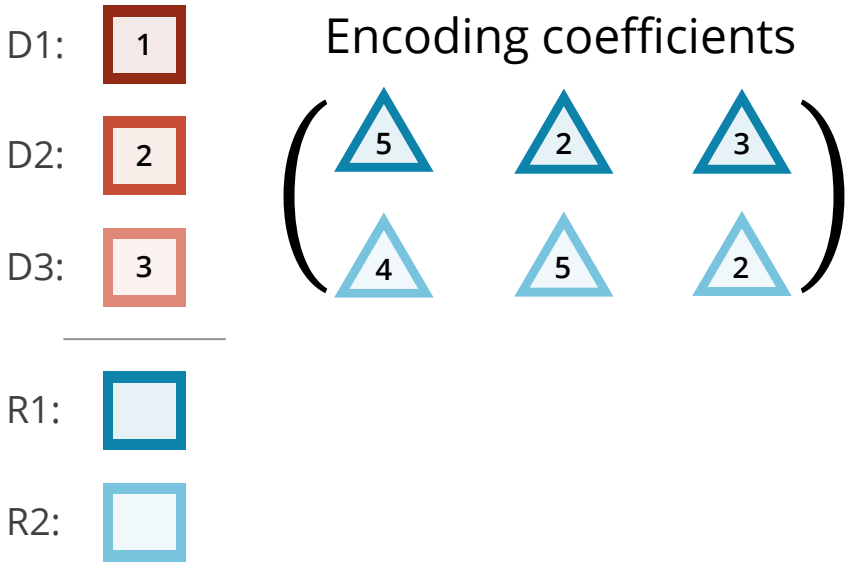
Fields

- Finite fields – p^q elements
- Solving linear equation systems works!
- Reed-Solomon: typically 2^q elements
  - $GF(2^8)$ – $[0,255]$

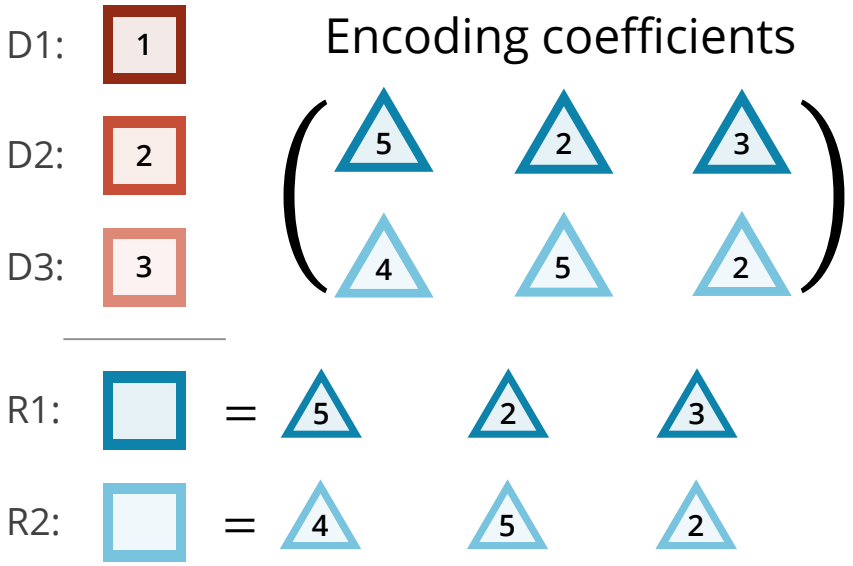
Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



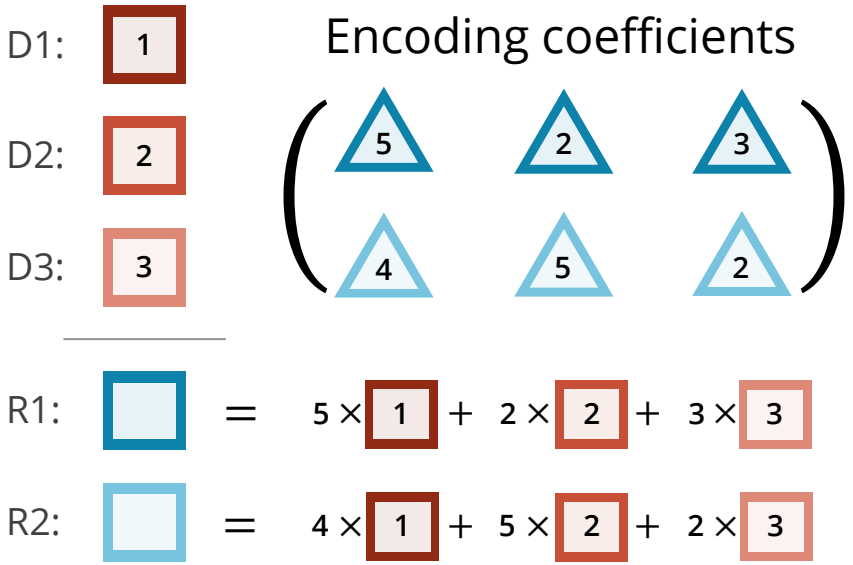
Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



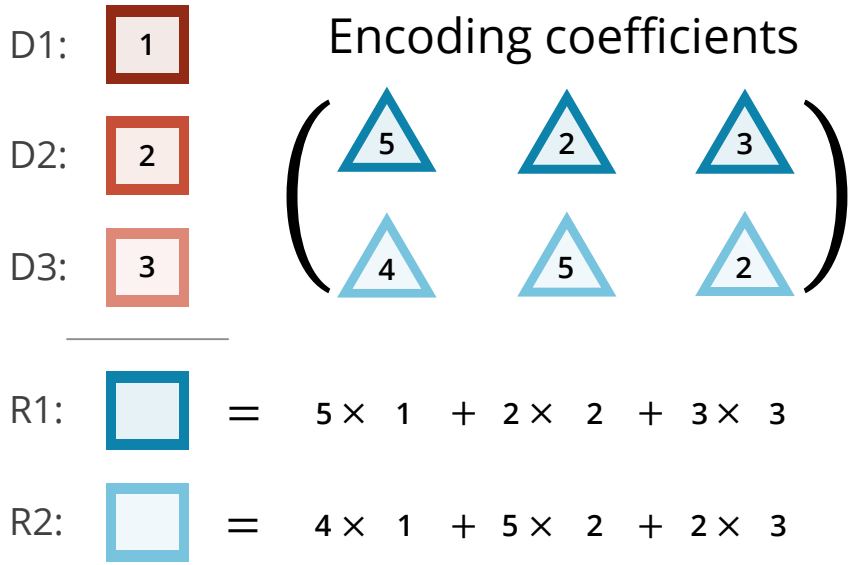
Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



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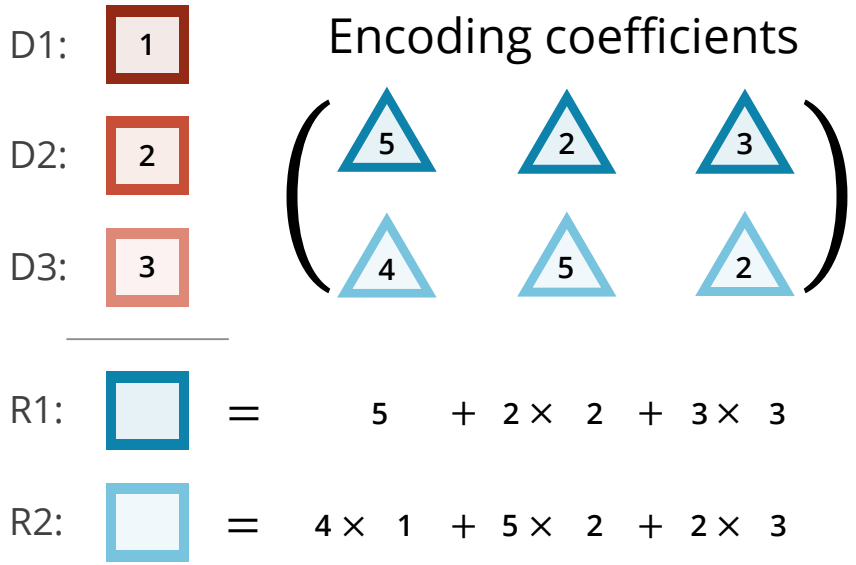
Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



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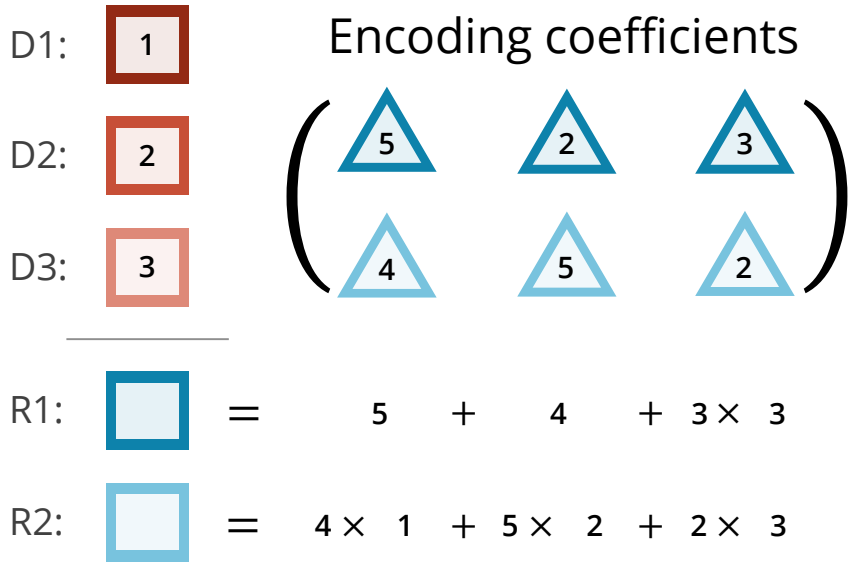
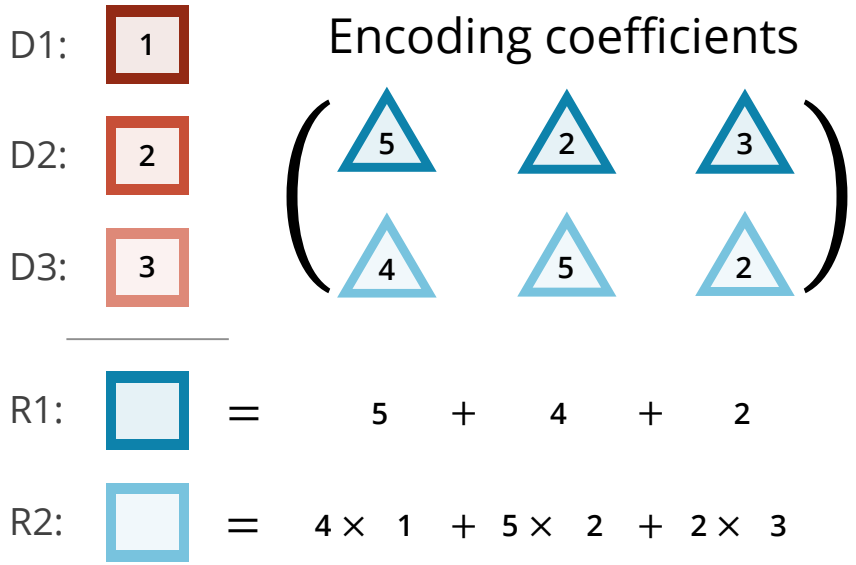
Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



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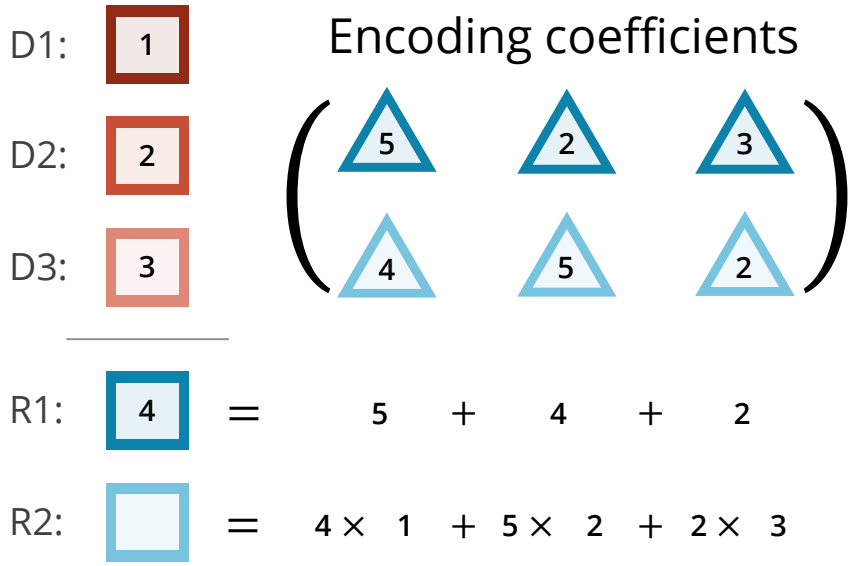
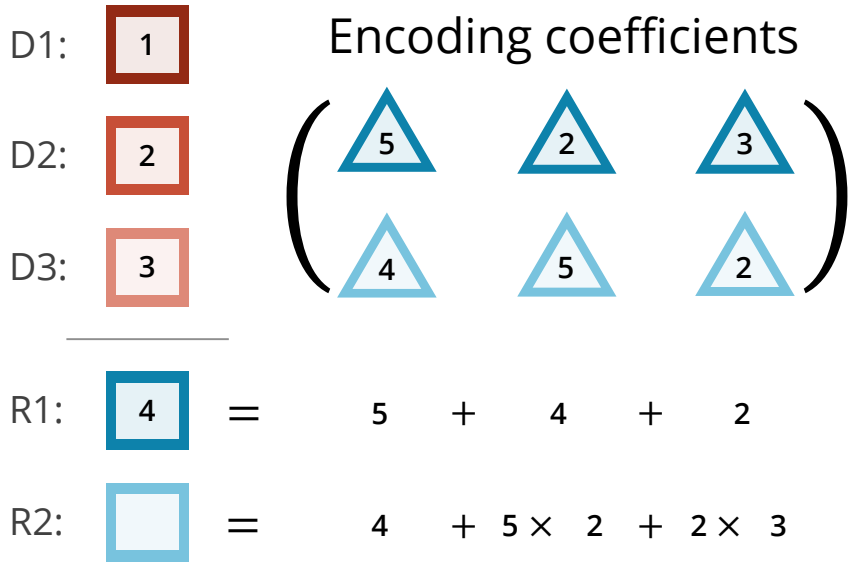


Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



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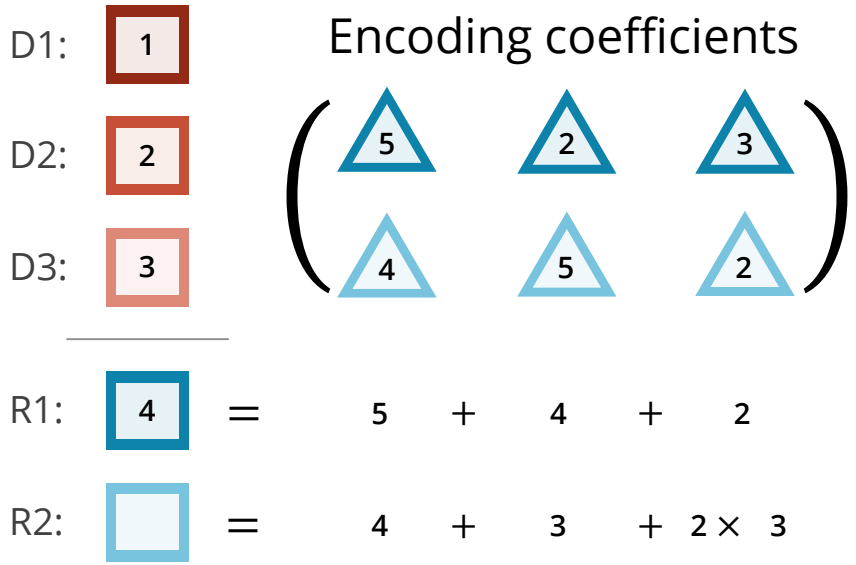
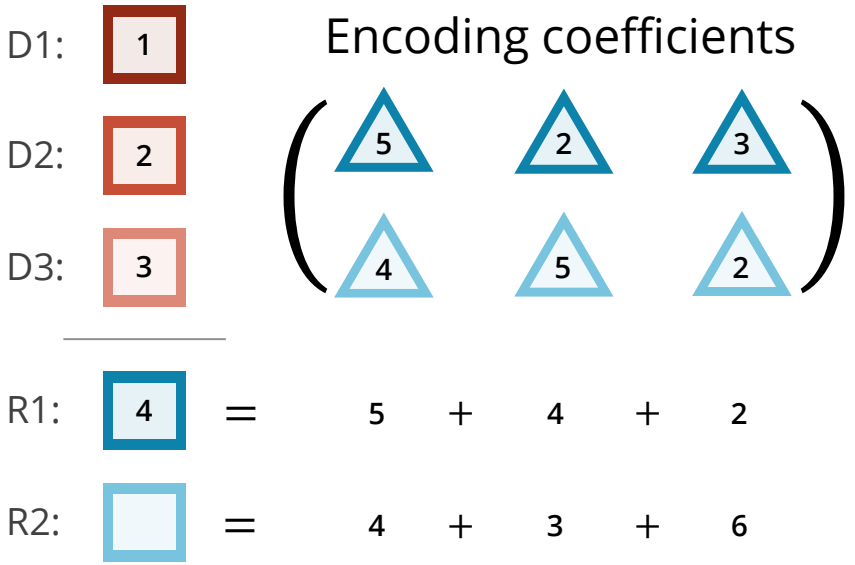


Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – encoding



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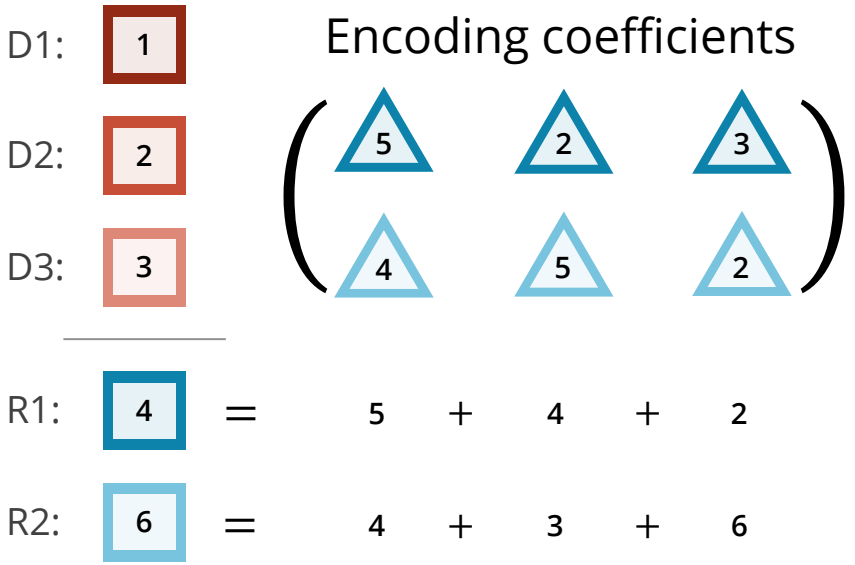

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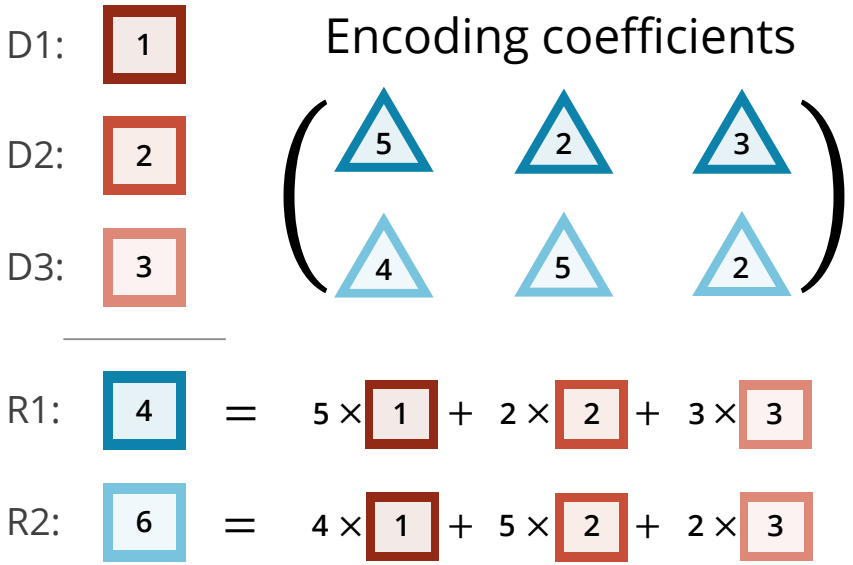
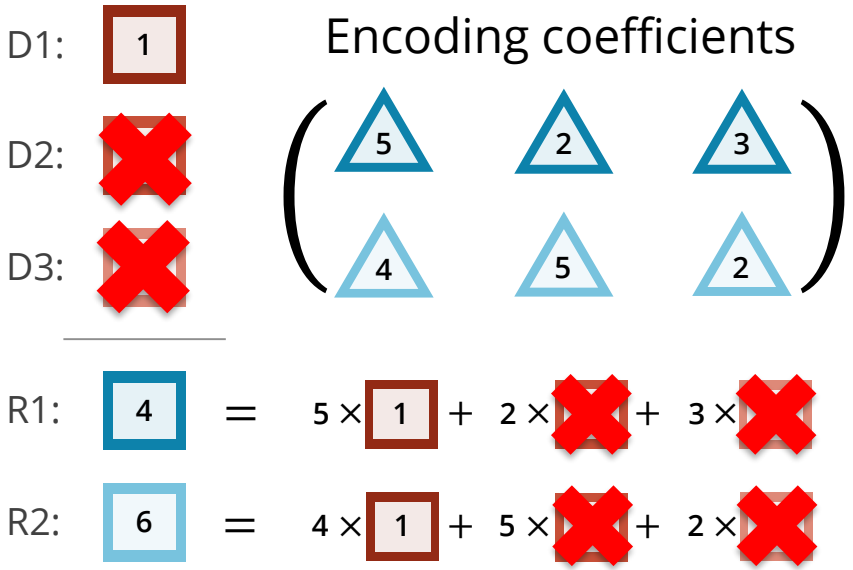
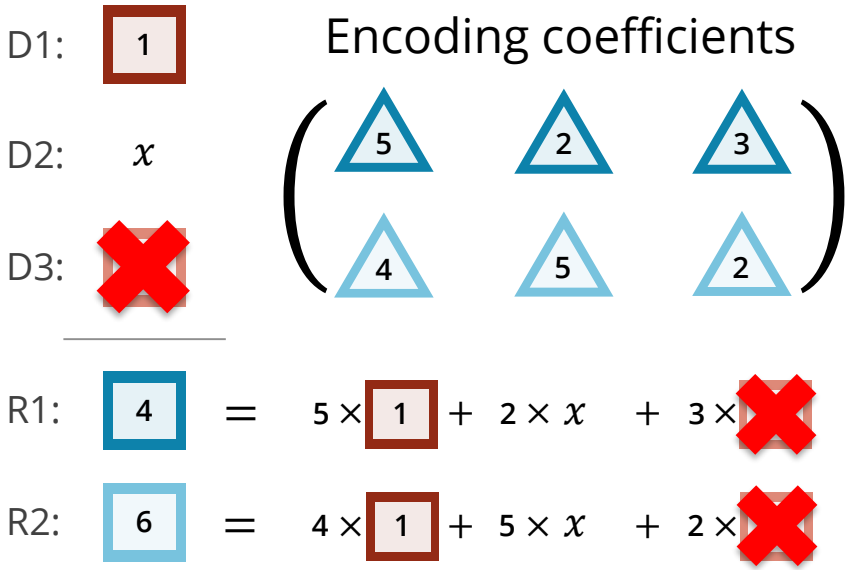
Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding



Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding



Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding



Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding

D1: 1

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 4 $= 5 \times 1 + 2 \times x + 3 \times y$

R2: 6 $= 4 \times 1 + 5 \times x + 2 \times y$

Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding

D1: 1

D2: x

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Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 4 = 5 × 1 + 2 × x + 3 × y

R2: 6 = 4 × 1 + 5 × x + 2 × y

$\Leftrightarrow$

$$5 + 2x + 3y = 4$$

$$4 + 5x + 2y = 6$$

Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding

D1: 1

D2: x

D3: y

Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: 4 $=$ $5 \times 1 + 2 \times x + 3 \times y$

R2: 6 $=$ $4 \times 1 + 5 \times x + 2 \times y$

$\Leftrightarrow$

$$2 + 5 + 2x + 3y = 4 + 2$$

$$3 + 4 + 5x + 2y = 6 + 3$$

Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding

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Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

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$\Leftrightarrow$

$$\cancel{2+5} + 2x + 3y = 4 + 2$$

$$\cancel{3+4} + 5x + 2y = 6 + 3$$

Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding

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R2: 6 $= 4 \times 1 + 5 \times x + 2 \times y$

$$\Leftrightarrow \quad \begin{aligned} 2x + 3y &= 4 + 2 \\ 5x + 2y &= 6 + 3 \end{aligned}$$

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D1: 1

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D3: y

Encoding coefficients

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$$\Leftrightarrow \quad \begin{aligned} 2x + 3y &= 6 \\ 5x + 2y &= 2 \end{aligned}$$

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D1: 1

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$$\Leftrightarrow \quad \begin{array}{l} 2x + 3y = 6 \\ 5x + 2y = 2 \\ \hline 0x + 5y = 1 \end{array}$$

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Multiplication table for $\mathbb{Z}/_{7\mathbb{Z}}$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2		4	6	1	3	5
3			2	5	1	4
4				2	6	3
5					4	2
6						1

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Linear MDS codes over $\mathbb{Z}/7\mathbb{Z}$ – decoding

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Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

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4				2	6	3
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R1: $4 = 5 \times 1 + 2 \times x + 3 \times y$

R2: $6 = 4 \times 1 + 5 \times x + 2 \times y$

$$\Leftrightarrow \quad \begin{array}{r} 2x + 3y = 6 \\ 5x + 2y = 2 \\ \hline 3 \cdot 5y = 1 \cdot 3 \end{array}$$

Multiplication table for $\mathbb{Z}/_{7\mathbb{Z}}$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2		4	6	1	3	5
3			2	5	1	4
4				2	6	3
5					4	2
6						1

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Linear MDS codes over $\mathbb{Z}/_{7\mathbb{Z}}$ – decoding

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Encoding coefficients

$$\begin{pmatrix} 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$$

R1: $4 = 5 \times 1 + 2 \times x + 3 \times y$

R2: $6 = 4 \times 1 + 5 \times x + 2 \times y$

$$\Leftrightarrow \quad \begin{aligned} 2x + 3y &= 6 \\ 5x + 2y &= 2 \\ \hline y &= 3 \end{aligned}$$

Multiplication table for $\mathbb{Z}/_{7\mathbb{Z}}$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2		4	6	1	3	5
3			2	5	1	4
4				2	6	3
5					4	2
6						1

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FEC it up!

- GStreamer elements
  - RED encoder and decoder
  - ULPFEC decoder (XOR and Reed-Solomon)
  - RTPStorage element
- Reed-Solomon encoder and decoder
  - Only $GF(2^8)$
  - Matrix operations
  - Cauchy-RS – fast matrix inversion
  - SIMD optimizations[1]

[1] Anvin, H. P. The mathematics of RAID-6. 2009.

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DEMO THYME

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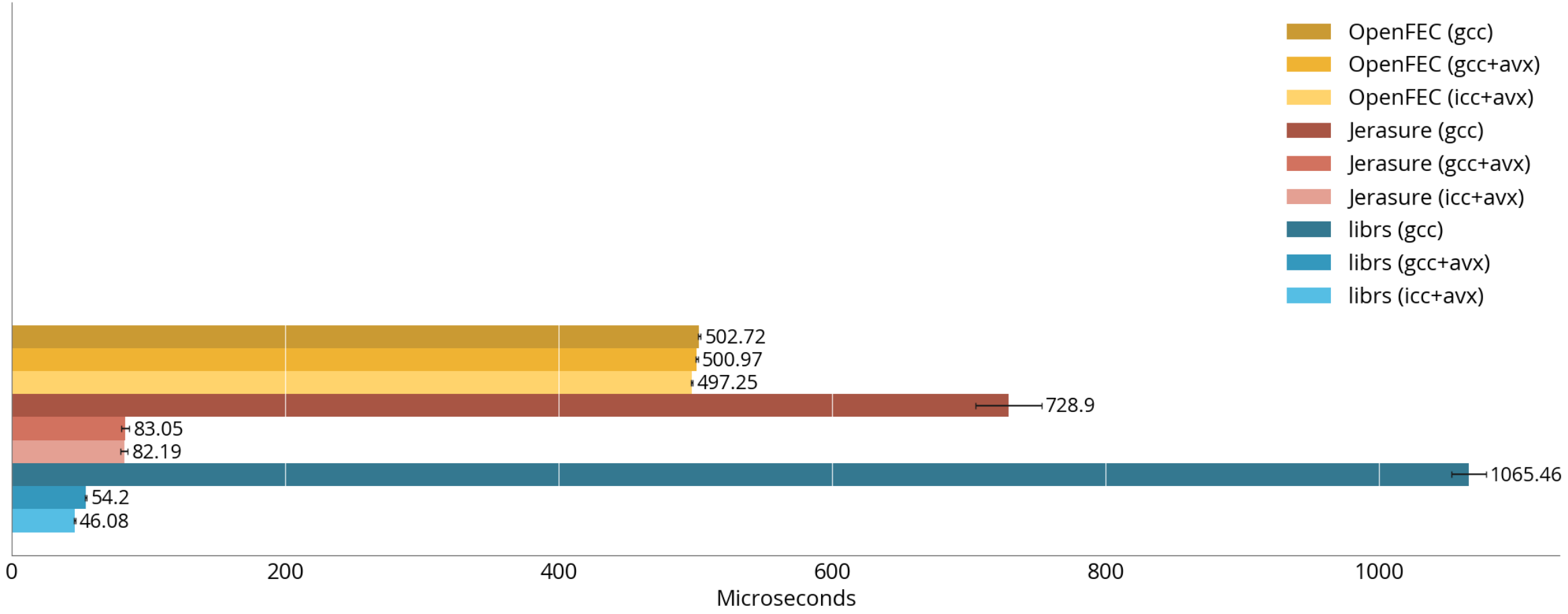
Benchmarks – Reed-Solomon encoding



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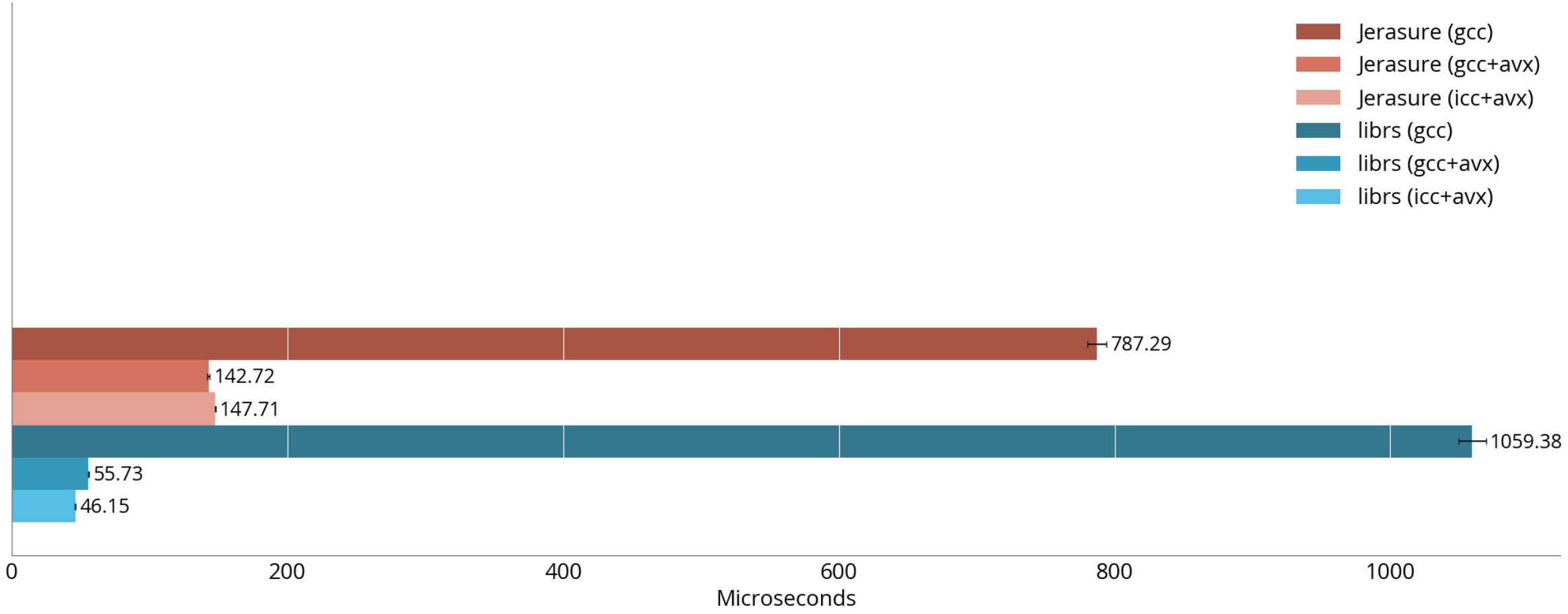
Benchmarks – Reed-Solomon decoding



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Questions?

(Do you think we can FEC?)

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